Markov Chain based PD Term Structure Modelling in an IFRS 9 Framework

MASTER’S THESIS

Submitted in partial fulfillment of the requirements for the degree of Master of Science in Banking and Finance

Author

THOMAS LUDWIG
Student-ID: 01415633
THOMAS.LUDWIG@student.uibk.ac.at

Supervisor

UNIV.-PROF. DR. MATTHIAS BANK, CFA
Department of Banking and Finance
University of Innsbruck

Date of Submission: August 2019
AFFIDAVIT

I declare that I have authored this thesis independently, that I have not used other than the declared sources/resources, and that I have explicitly marked all material which has been quoted either literally or by content from the used sources.

.................................................  .................................................
date  (signature)
This master’s thesis addresses the quantitative aspect of PD term structure modelling in an IFRS 9 framework. By using credit exposure data from the Italian banking sector, the thesis builds a comprehensive quantitative analysis based on a time homogeneous Markov chain approach for modelling the PD term structure of the private and corporate sector in Italy. The results show, that the assumption of time homogeneity is critical. Therefore using the average matrix for the PD term structure estimation does not fulfill all IFRS 9 requirements for PD modelling. We propose to calibrate the model on the most recent migration matrix for better compliance. Also in this case, the predictive power is likely to be poor after three to four years. As consequence, we stress the need for model validation and model risk management in the context of IFRS 9. Finally we propose approaches to generalize the PD modelling for IFRS 9 purposes for other applications to provide a coherent PD modelling framework.

**Keywords:** IFRS 9, PD modelling, Markov chain modelling, PD Term Structure, Credit Risk Modelling.
ACKNOWLEDGEMENTS

First and foremost I would like to thank everybody who has helped and supported me over the past five years during my studies in Innsbruck and Göteborg. In particular I would like to express gratitude to Professor Matthias Bank who provided valuable guidance and who gave me the opportunity to be at the Department of Banking and Finance throughout the development of this thesis. Furthermore I thank my classmates Kevin Kraemer, Felix Kunz and Thomas Pichler for continuous discussions and valuable inputs for this work during our last semester in Innsbruck.

Additionally I thank Stephan Oberarzbacher for giving me the chance to work part-time in a very interesting position and gain priceless practical and personal experiences besides my studies for this degree over the last two years.

Lastly I thank my family for continuous support and encouragement.
Contents

List of Figures III

List of Tables V

List of Abbreviations VI

1 Introduction 1

2 Theoretical Background 3

2.1 Credit Risk in Theory and Practice . . . . . . . . . . . . . . . . . 3

2.2 Mathematical Processes to Describe Credit Risk . . . . . . . . . . 7

2.2.1 Components of Credit Portfolio Loss . . . . . . . . . . . . . . 7

2.2.2 Credit Rating and 12-month PD Estimation . . . . . . . . . . 11

2.2.3 Migration Matrices and PD Term Structure . . . . . . . . . . 14

2.2.4 Expected Credit Loss and Loan Pricing . . . . . . . . . . . . 16

2.3 Credit Risk and IFRS 9 . . . . . . . . . . . . . . . . . . . . . . . 18

2.3.1 IFRS 9 in a Nutshell . . . . . . . . . . . . . . . . . . . . . . . 18

2.3.2 Impairment and Expected Credit Loss Model . . . . . . . . . 20

2.3.3 PD Modelling Requirements in IFRS 9 . . . . . . . . . . . . . 24

3 Markov Chain based PD Term Structure Modelling 28

3.1 Markov Chain Theory and PD Modelling . . . . . . . . . . . . . . 28

3.2 Markov Chain Models for IFRS 9 PD Modelling . . . . . . . . . . 35

3.3 Parameter Estimation and Model Validation . . . . . . . . . . . . 37

3.3.1 Transition Matrix Estimation . . . . . . . . . . . . . . . . . . 37

3.3.2 Model Validation for IFRS 9 PD Models . . . . . . . . . . . . 39

3.3.3 Analysis of Time Inhomogeneity . . . . . . . . . . . . . . . . 41
3.3.4 Uncertainty Quantification with Confidence Intervals

4 Empirical Analysis

4.1 Data Description and Descriptive Analysis
4.2 Cure Rate and Danger Rate Estimation
4.3 Recovering Defaults in a Markov Chain Framework
4.4 PD Term Structure Estimation
  4.4.1 Cumulative PD Term Structure Estimation
  4.4.2 Marginal PD Calculation
4.5 Model Validation
  4.5.1 Testing for Time Inhomogeneity
  4.5.2 Estimation of Recession and Expansion Matrices
  4.5.3 PD Confidence Intervals
4.6 Discussion of Results in Context of IFRS 9

5 Discussion and a View beyond IFRS 9

5.1 Expected Credit Loss Mispricing - How Accurate can Expected Credit Loss Modelling be?
5.2 Impact of IFRS 9 PD Modelling on Banking Processes
5.3 Aspects of Validation and IT Architecture in a Coherent PD Modelling Framework
5.4 Suggestions for Future Research

6 Conclusion

Bibliography
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Historical Default Rates Comparison in Italy Private and Corporate Sector</td>
<td>6</td>
</tr>
<tr>
<td>2.2</td>
<td>Probability Density Function (PDF) of $D_{i,t}$ for a Credit Portfolio with Default Correlation</td>
<td>8</td>
</tr>
<tr>
<td>2.3</td>
<td>Expected Exposure Profile of Financial Instruments (Fixed-rate bond/amortizing loan)</td>
<td>9</td>
</tr>
<tr>
<td>2.4</td>
<td>Illustration of Default Intensity $\lambda$ for PD representation in continuous time based on Herbertsson (2018, p. 44)</td>
<td>11</td>
</tr>
<tr>
<td>2.5</td>
<td>Logistic Function $f(x) = \frac{1}{1+\exp^{-x}}$ used in 12-month PD estimation</td>
<td>13</td>
</tr>
<tr>
<td>2.6</td>
<td>Empirical PD Term Structure Global Corporate Average Cumulative Default Rates (1981-2018) (Data from Standard &amp; Poor’s RatingsDirect (2018, Table 24))</td>
<td>16</td>
</tr>
<tr>
<td>2.7</td>
<td>IFRS 9 Bucket Approach for Stage Allocation</td>
<td>23</td>
</tr>
<tr>
<td>3.1</td>
<td>Trajectory from Simulated Markov Chain with State Space $E = {1,2,3}$</td>
<td>31</td>
</tr>
<tr>
<td>3.2</td>
<td>Visualization of Markov Chain Transition Matrix with State Space $E = {\text{AAA, BBB, D}}$</td>
<td>32</td>
</tr>
<tr>
<td>3.3</td>
<td>PD Term Structure and Survival Rate in a Markov Chain Model</td>
<td>34</td>
</tr>
<tr>
<td>3.4</td>
<td>Marginal PD Structure in a Markov Chain Model</td>
<td>35</td>
</tr>
<tr>
<td>4.1</td>
<td>NPE Analysis Private Sector</td>
<td>48</td>
</tr>
<tr>
<td>4.2</td>
<td>NPE Analysis Corporate Sector</td>
<td>48</td>
</tr>
<tr>
<td>4.3</td>
<td>Cure Rate Comparison 2011–2018</td>
<td>50</td>
</tr>
<tr>
<td>4.4</td>
<td>Danger Rate Comparison 2011–2018</td>
<td>51</td>
</tr>
<tr>
<td>4.5</td>
<td>Estimated PD Term Structure for Private and Corporate Sector</td>
<td>55</td>
</tr>
<tr>
<td>4.6</td>
<td>Marginal PD for Private and Corporate Sector</td>
<td>56</td>
</tr>
</tbody>
</table>
List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>2018 One-Year Global Corporate Transition Rate (%) (Standard &amp; Poor’s RatingsDirect, 2018 Table 20)</td>
<td>14</td>
</tr>
<tr>
<td>2.2</td>
<td>Global Corporate Average Cumulative Default Rates 1981-2018 (%) (Standard &amp; Poor’s RatingsDirect, 2018 Table 24)</td>
<td>15</td>
</tr>
<tr>
<td>2.3</td>
<td>Financial Instrument Classification in IFRS 9</td>
<td>20</td>
</tr>
<tr>
<td>4.1</td>
<td>Variable Summary</td>
<td>46</td>
</tr>
<tr>
<td>4.2</td>
<td>Migration Matrix Corporate Sector 2017–2018 (Mio. EUR)</td>
<td>47</td>
</tr>
<tr>
<td>4.3</td>
<td>Cure Rate and Danger Rate Corporate Sector 2017–2018 (Mio. EUR)</td>
<td>50</td>
</tr>
<tr>
<td>4.4</td>
<td>Average Migration Matrix Private 2011–2018 (%)</td>
<td>53</td>
</tr>
<tr>
<td>4.5</td>
<td>Average Migration Matrix Corporate 2011–2018 (%)</td>
<td>53</td>
</tr>
<tr>
<td>4.6</td>
<td>Cumulative PD Term Structure Estimates</td>
<td>54</td>
</tr>
<tr>
<td>4.7</td>
<td>Marginal PD Estimates</td>
<td>55</td>
</tr>
<tr>
<td>4.8</td>
<td>Recession and Expansion Matrices Cumulative PD</td>
<td>61</td>
</tr>
<tr>
<td>4.9</td>
<td>Recession and Expansion Matrices Marginal PD</td>
<td>61</td>
</tr>
<tr>
<td>4.10</td>
<td>PD Estimates and Confidence Intervals Private Sector</td>
<td>64</td>
</tr>
<tr>
<td>4.11</td>
<td>PD Estimates and Confidence Intervals Corporate Sector</td>
<td>64</td>
</tr>
</tbody>
</table>
# List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviations</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>Amortized Cost</td>
</tr>
<tr>
<td>CR</td>
<td>Centrale dei Rischi</td>
</tr>
<tr>
<td>DR</td>
<td>Default Rate</td>
</tr>
<tr>
<td>e.g.</td>
<td>exempli gratia (“for the sake of an example”)</td>
</tr>
<tr>
<td>EAD</td>
<td>Exposure at Default</td>
</tr>
<tr>
<td>EBA</td>
<td>European Banking Authority</td>
</tr>
<tr>
<td>ECB</td>
<td>European Central Bank</td>
</tr>
<tr>
<td>ECL</td>
<td>Expected Credit Loss</td>
</tr>
<tr>
<td>ECLM</td>
<td>Expected Credit Loss Model</td>
</tr>
<tr>
<td>ECP</td>
<td>Expected Credit Premium</td>
</tr>
<tr>
<td>ELR</td>
<td>Expected Loss Rate</td>
</tr>
<tr>
<td>ES</td>
<td>Expected Shortfall</td>
</tr>
<tr>
<td>EU</td>
<td>European Union</td>
</tr>
<tr>
<td>FVTOCI</td>
<td>Fair Value through Other Comprehensive Income</td>
</tr>
<tr>
<td>FVTPL</td>
<td>Fair Value through Profit and Loss</td>
</tr>
<tr>
<td>GDP</td>
<td>Gross Domestic Product</td>
</tr>
<tr>
<td>IASB</td>
<td>International Accounting Standards Board</td>
</tr>
</tbody>
</table>
ICAAP Internal Capital Adequacy Assessment Process
IFRS International Financial Reporting Standards
ILAAP Internal Liquidity Adequacy Assessment Process
LGD Loss Given Default
NPE Non Performing Exposure
NPE Ratio Non Performing Exposure Ratio
NPL Non Performing Loan
PD Probability of Default
PD 90 90-days Past Due
PIT Point-in-Time
QRM Quantitative Risk Management
RR Recovery Rate
Soff Sofferenze
TTC Through-the-Cycle
UTP Unlikely to Pay
VaR Value at Risk

**Selected Mathematical Symbols**

\((X_t)_{t \in T}\) Stochastic Process
\(\alpha\) Initial Distribution of Stochastic Process
\(E\) State Space
\(P\) Transition Matrix
\(\Delta M_{L2}\) Distance between two Matrices
\(\Delta t\) Discrete Time Step
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{E}[x]$</td>
<td>Expectation of a Random Variable $x$</td>
</tr>
<tr>
<td>$\hat{P}$</td>
<td>Transition Matrix Estimate</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Normal Distribution Quantile</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Default Intensity</td>
</tr>
<tr>
<td>$\Omega, \mathcal{F}, P$</td>
<td>Probability Triplet</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Default Time</td>
</tr>
<tr>
<td>$PD^P$</td>
<td>Real-world PD</td>
</tr>
<tr>
<td>$PD^Q$</td>
<td>Risk-neutral PD</td>
</tr>
<tr>
<td>$D_i$</td>
<td>Default Indicator Variable</td>
</tr>
<tr>
<td>$L_2$</td>
<td>Matrix Norm</td>
</tr>
<tr>
<td>$n_{ij}$</td>
<td>Number of Firms</td>
</tr>
<tr>
<td>$r_i$</td>
<td>Discount Rate</td>
</tr>
<tr>
<td>$V[x]$</td>
<td>Variance of the Random Variable $x$</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>PMF</td>
<td>Probability Mass Function</td>
</tr>
<tr>
<td>r.v.</td>
<td>Random Variable</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

The mandatory introduction of the IFRS 9 standard published by the International Accounting Standards Board (IASB) starting with 1st January 2018 had a significant impact on the banking industry on various levels. The new standard introduced a paradigm shift towards a new approach for the impairment loss recognition in a bank’s balance sheet. IFRS 9 is a more principle based standard, hence the standard defines broad guidelines, and does not define specific methodological approaches in many cases. As consequence of this approach, the actual implementation method is in the responsibility of the individual bank.

The expected credit loss model (ECLM) is in the center of the new approach for the impairment accounting of financial instruments. This model comes along with the need for quantitative modelling of its input parameters, which are the probability of default (PD), the exposure at default (EAD) and the loss given default (LGD). Academia and practitioners from various research areas were fast to propose various modelling approaches, which aim to comply with the principles defined by the standard setter.

This master’s thesis aims to assess one specific parameter of the expected credit loss model, namely the probability of default (PD). In the context of IFRS 9 this parameter needs to fulfill a range of specific requirements which require sophisticated modelling approaches. In this work we investigate one specific modelling approach, which is widely used in the industry, namely a time homogeneous
Chapter 1. Introduction

Markov chain based modelling approach. From this rather general motivation, the research questions are concretized in the following way.

“To what extend is a time homogeneous Markov chain based modelling approach appropriated for the PD term structure modelling from an methodological and IFRS 9 compliance point of view?”

“How robust is a time homogeneous Markov chain based implementation with respect to the input parameters, what are appropriate approaches to quantify the parameter estimation uncertainty and how can the results be validated?”

The operationalization of the defined research questions is carried out by firstly providing a comprehensive overview and introduction about credit risk and quantitative modelling. Furthermore, the first section provides a introduction towards IFRS 9 with focus on impairment accounting.

The second section is devoted to the use of Markov chain theory for modelling the PD term structure with a methodological introduction to Markov chain processes. Furthermore, the bridge towards the implementation in the IFRS 9 context is built by relating to recent literature, and by providing the theoretical framework for the use of a Markov chain PD modelling. Special emphasis is on the aspect of model validation, verification of model assumptions and the robustness of the results.

The empirical part of this thesis is performed by the application of the theoretical framework to data retrieved from Banca d’Italia, covering credit exposure held by the Italian banking sector for the private and corporate sector in Italy. In this setting a quantitative analysis to answer the defined research questions is presented. The PD term structure and all needed quantities for the IFRS 9 application are calculated and discussed. Furthermore, the model assumptions and the robustness of the results are investigated by applying the methodological framework.

The thesis concludes with a critical discussion of the results, and by aiming to generalize the IFRS 9 PD modelling framework to other applications within a bank.
Chapter 2

Theoretical Background

This section introduces fundamental credit risk management concepts with emphasis to the application in an IFRS 9 standard framework. A mathematical description of risk in general, credit risk, and regulatory requirements are introduced. Furthermore, an introduction in the IFRS 9 standard and the expected credit loss model (ECLM) and its parameters are presented. The section concludes with presenting PD modelling approaches in IFRS 9 discussed by recent literature.

2.1 Credit Risk in Theory and Practice

McNeil et al. (2005, pp. 1–4) introduce the concept of risk by referring to the definition presented in the Oxford English Dictionary, which defines risk as “hazard, a change of bad consequences, loss or exposure to mischance”. Financial risk management can be distinguished in general within the risk categories market risk, credit risk, operational risk, liquidity risk and model risk. In this work the risk category of interest is credit risk. See McNeil et al. (2005) for a presentation of all risk categories from a mathematical finance perspective. In this section we use the notation presented in McNeil et al. (2005), which use a rigorous mathematical approach towards risk modelling, which is in the book summarized under the term quantitative risk management (QRM).
Definition 2.1.1. Credit or default risk. As defined by McNeil et al. (2005, p. 327) in the most general terms default risk is defined as the risk that an obligor does not honor his obligations. In financial institutions this obligations are mostly referred to loans or bonds. The risk of payment default by the counterparty is in this context referred to as credit risk

Herbertsson (2018, p. 2) stresses that credit risk literature proposes the following risk decomposition which will be important for the next sections of this master’s thesis:

- **Arrival risk**: Describes the risk that an obligor will default, or not in a defined time-period (e.g. one year). This risk is typically quantified by estimating a probability of default (PD) for an individual obligor in a larger credit portfolio.

- **Timing risk**: Describes the risk which is connected with the timing of arrival risk.

- **Recovery risk**: Describes the risk which is connected to the actual loss for a bank in a default event. This risk is typically quantified by using the exposure at default (EAD) and the loss given default (LGD), which is the counterpart of the recovery rate (RR).

- **Default dependency risk/portfolio credit risk**: Describes the risk, that default events in a bank’s portfolio happen jointly during certain time periods ("default correlation and default dependence"). This risk is quantified by using models considering the credit portfolio framework.

Herbertsson (2018 pp. 1–2) indicates that in credit risk management a financial institutions obligor can be a company that has received money from the bank or a company that has issued fixed-income securities (e.g. coupon bonds). On the other hand, obligors can also be private persons and households if they get a loan to buy a house or a car.

From this rather abstract definition we become aware of the basic definition. However, in practical terms the definition of credit risk depends heavily on the
Chapter 2. Theoretical Background

concept of default definition. In general terms, a default event for the above mentioned examples would be if the company that borrowed money from a bank went bankrupt, or if the company that issued bonds fails to pay the coupon on time. For the private household a default could be defined when the household fails to pay the agreed amount for the repayment of their loan. The definition of the default event from a legal perspective is a very important aspect in a bank’s and regulators credit risk management. In this thesis the default definition of Italy is presented.

Italian banks are required to classify positions in their credit portfolio since 2014 as non-performing exposures (“notleidende Risikopositionen”) or performing exposures (“vertragsmäßig bediente Risikopositionen”). The classification was introduced on European level by the European Banking Authority (2013, Art. 178), which extends the national default definition for Italy introduced by the regulatory requirements in Banca d’Italia (2008) and subsequent extensions published by Banca d’Italiy (Bank of Italy). The default definition including the German and Italian denomination is given by:

- **Sofferenze** (“Zahlungsunfähige Kredite“): Positions that are not able to meet their payment obligations where the key criteria for the determination is the inability to meet payment obligations.

- **Unlikely to pay** (“Kredite mit wahrscheinlichem Zahlungsausfall”/“Inadempienze probabili“): Positions for that it is unlikely that the subject is able to meet their full payment obligations without the use of collateral by the bank. It is not required that a position has already missed a payment to classify this default category as non-performing exposure.

- **Material exposures which are more than 90 days past due** (“Überfällige Kredite > 90 Tage”/“Esposizioni scadute e/o sconfiniti deteriorate“): Positions not already classified as another default state, which did not meet their payments for a continuous time of at least 90 days.

The default classification as *sofferenze* exists only in Italy, therefore no adequate English name is available. In the empirical part of this thesis the original names
for the default definition are used. Global Public Policy Committee [2016, p. 26] note, that IFRS 9 does not define the default event. Each financial institution is required to use a default definition, which is consistent with the default definition used in internal credit risk management. Following this, the in this thesis presented default definitions are also used for IFRS 9.

Figure 2.1: Historical Default Rates Comparison in Italy Private and Corporate Sector

Figure 2.1 presents the empirical default rates in overall Italy and the autonomous province of South Tyrol for the private and corporate sector, which are typically collected and elaborated by the national central bank in the Italian case of Banca d’Italia (Bank of Italy). Important to note is the aspect of different default definitions over time, which has to be kept in mind when comparing default rates over time.

The graph indicates a cyclical behavior of the default rate over time. Numerous studies, such as Bonfim [2009] show a relationship between the credit quality in bank’s portfolios and the respective macroeconomic environment. Hence, the default rate “follows” the economic cycle including recessions with higher default rate and periods of expansions with lower default rate. As indicated by McNeil et al. [2005, p. 349], it is common in credit risk modelling to split the determinants that drive the credit quality of firms or households in systematic and idiosyncratic factors. The systematic factor can be interpreted as the macroeconomic variables (e.g. unemployment rate, GDP growth), which have an impact on the overall credit quality of all firms.
Chapter 2. Theoretical Background

2.2 Mathematical Processes to Describe Credit Risk

Credit risk modelling is as any risk modelling objective to dealing with risk and randomness. McNeil et al. (2005, pp. 1–3) show the need to introduce a mathematical notion for randomness in order to mathematically describe risk. In the most basic notation, the probability triple \((\Omega, \mathcal{F}, P)\) describes a probabilistic model. An element \(\omega\) of \(\Omega\) is defined as the realization of an experiment. As a result, one can define the probability that event \(A\) occurs as \(P(A)\), where \(A \in \mathcal{F}\) and \(\mathcal{F}\) is the set of all events. \(P\) is defined as the probability measure. If \(X\) is defined as function on the probability space \((\Omega, \mathcal{F}, P)\), the function is named as random variable.

In order to formally describe credit risk, we make use of the formally introduced concept of randomness. Credit risk management in financial institutions is typically dealing with risk on a portfolio level. In order to describe a bank’s credit portfolio, we define a credit portfolio with \(N\) obligors, each with index \(i\), where \(i = \{1, 2, \ldots, N\}\). The default time is a random variable \(\tau_i\) and varies across the obligors in the portfolio. With this setting it is possible to formally describe the bank’s portfolio loss by introducing the main components of the individual loss and aggregating it to the total portfolio loss.

2.2.1 Components of Credit Portfolio Loss

In this section the components of portfolio loss are presented following derivation and the notation presented in Schmidt (2016) and McNeil et al. (2005). Furthermore, it is discussed how they are used for the computation of the expected loss of a credit portfolio. The first and for the scope of this thesis most important risk parameter is the default indicator process \(D_{i,t}\) which will be related to the PD of a position in a credit portfolio.

Definition 2.2.1. Default indicator process. \(D_{i,t}\) is a indicator function which is defined as:
Chapter 2. Theoretical Background

\[ D_{i,t} := \begin{cases} 1 & \text{if obligor } i \text{ defaults in } t, \\ 0 & \text{if obligor } i \text{ has not defaulted in } t. \end{cases} \]

where \( i \in N_t \), \( t = \{1, 2, \ldots, T\} \). In this notation \( N_t \) is defined as the obligors which did not default at the start of \( t \). We define the interval \([1, T]\) as the PD estimation period. We recall that an indicator random variable is a bridging function between probability and expectation. In this thesis the probability of default is denoted as PD and refers typically to the 12-month PD as this is the typical time horizon in credit risk management.

The default indicator follows therefore a Bernoulli distribution. Hence \( D_i \sim \text{Ber}(PD_i) \), where \( PD_i = P(D_i = 1) = E[D_i] \). Therefore, it follows that PD \( \in [0, 1] \). This notation links the expectation of the default indicator process (which is a random variable) to the probability of default since for indicator functions it holds that \( E(D_{i,t}) = P(A) \). The process \( D_i \) can be characterized in continuous time by its distribution (cumulative distribution function (CDF) and probability density function (PMF).

![Probability Density Function (PDF) of \( D_{i,t} \) for a Credit Portfolio with Default Correlation](image)

**Figure 2.2:** Probability Density Function (PDF) of \( D_{i,t} \) for a Credit Portfolio with Default Correlation
As noted by McNeil et al. (2005, p. 330), the shape in Figure 2.2 is typical for a loss distribution of a credit portfolio where the default events are characterized by default dependence as it is the typical case (the default event is not independent among the obligors, but they are correlated). The default dependency generates the heavy tail of the distribution which increases tail risk. Modelling default dependencies is the typical task of credit portfolio risk (Herbertsson, 2018, pp. 72–131).

**Definition 2.2.2.** Exposure at default (EAD) process $EAD_{i,t}$. The exposure at default is the total amount outstanding at time $t$. Exposure profiles of financial instruments can be deterministic or stochastic. If we consider a loan with a prepayment option, the exposure profile is depending on a set of variables and is therefore not deterministic (Schmidt, 2016).

![Expected Exposure Profile of Financial Instruments (Fixed-rate bond/amortizing loan)](image)

**Figure 2.3:** Expected Exposure Profile of Financial Instruments (Fixed-rate bond/amortizing loan)

**Definition 2.2.3.** Loss given default (LGD) process $LGD_{i,t}$. The LGD process $LGD_{i,t}$ is defined as the fraction which is lost after the default occurred. Therefore the LGD takes a value between zero and one, hence $LGD \in [0, 1]$. The complement of the LGD is the recovery rate (RR), which is denoted as $1 - LGD$. Note, that LGD can be treated as stochastic variable. For this work we will not model the LGD in a stochastic way as we focus on the PD (Schmidt, 2016).
Chapter 2. Theoretical Background

The loss parameters can be used to compute the loss distribution of a credit portfolio. McNeil et al. (2005) indicate, that the loss distribution in a portfolio can typically be separated into the component of Expected Portfolio Loss and Unexpected Portfolio Loss. Modelling the loss distribution allows for example to compute commonly known risk measures as Value at Risk (VaR) or Expected Shortfall (ES) also in the context of credit risk.

In this section we will focus our analysis on the expected portfolio loss. With the above notation we follow the derivation of the expected portfolio loss presented in Schmidt (2016) as one can write the individual loss $L_{i,T}$ as function of the input parameters.

\[ L_{i,T} = D_{i,T} \times EAD_{i,\tau_i} \times LGD_{i,\tau_i} \]  \hspace{1cm} (2.1)

We can further aggregate the individual default loss for a portfolio of obligors.

\[ L_T = \sum_{i=1}^{N} L_{i,T} \]  \hspace{1cm} (2.2)

If we use the fact that the probability of default is the expectation of the default indicator process and we assume LGD and EAD to be constant, we can calculate the expectation for the 12-month expected credit loss for an obligor $i$ in the credit portfolio as:

\[ \mathbb{E}[L_{i,T}] = p_{i,T} \times EAD_i \times LGD_i \]  \hspace{1cm} (2.3)

For a portfolio with $N$ obligors the expected loss can be written as:

\[ \mathbb{E}[L_T] = \sum_{i=1}^{N} \mathbb{E}[L_{i,T}] \]  \hspace{1cm} (2.4)

Equation 2.4 will be used to link the well known concept of expected credit loss modelling to IFRS 9. As pointed out by Eder (2016, p. 19), expected credit loss models are widely used. One example is the Basel III context.
2.2.2 Credit Rating and 12-month PD Estimation

Before turning the focus on the estimation of a multi-period PD term structure, we start by presenting the basic estimation procedure for the 12-month PD, which is the base and therefore important. Baesens et al. (2016, p. 139) discuss the main types of PD measures according to the following classification:

- **Real-world or physical PD estimation**: The PD is modelled for a real-world default realization. This is mostly the case for loans given to borrowers (commonly referred to as $P$ measure, where the real world probability of default is denoted as PD$^P$).

- **Risk-neutral PD estimation**: The PD is derived from observed market prices (e.g. share price or derivative instruments as credit default swaps). Therefore, the PD is limited for companies with traded securities (commonly referred to as $Q$ measure, where the risk-neutral probability of default is denoted as PD$^Q$).

The PD used in this thesis is the physical measure, where we omit the PD$^P$ notation and use PD instead. Furthermore, the PD can be estimated in a discrete or continuous time setting. In continuous time the default intensity $\lambda$ is a crucial parameter.

![Figure 2.4: Illustration of Default Intensity $\lambda$ for PD representation in continuous time based on Herbertsson (2018, p. 44)](image.png)

Figure 2.4 describes the concept of default intensity. The flow of known information is denoted by filtration $\mathcal{F}$. Given $\mathcal{F}_t$ the default event $\tau$ will arrive in the interval $[t, t + \Delta t]$ with probability $\lambda \Delta t$, where $\lambda$ denotes the default intensity for the given process. Therefore, $\lambda$ is the instantaneous probability of default. Herbertsson (2018) indicates, that the continuous probability of default is often used for pricing financial contracts (e.g. Credit Default Swaps).

Next, we will show the relationship between default intensity $\lambda$ and PD by time $t$ as presented in Hull (2010, pp. 303–307):
Chapter 2. Theoretical Background

\[ PD_{0,t} = 1 - \exp^{- \int_0^t \lambda(\tau) d\lambda} \quad (2.5) \]

In contrast to the continuous time, the discrete PD quantifies the default probability of an obligor for a discrete time horizon. As noted by Rösch and Scheule (2007), the discrete PD is typically estimated for a real world default realization, so \( PD_{0,t} \) is the PD in \( t = 0 \) until \( t \), where in credit risk management the considered time horizon is typically one year. Note that in this thesis we work in a discrete time setting when estimating the PD, hence \( t = \{1, 2, \ldots, T\} \).

The main focus of the empirical analysis in this thesis is built upon rating migration matrices. Therefore it is worthwhile to analyze the basic properties of a credit rating model in more detail. The goal of a rating model is to separate “good’ from “bad” borrowers in a systematic way (Baesens et al., 2016, p. 93). A typical rating system is built on a mixture of quantitative and qualitative reasoning.

The quantitative part uses various logistic regression models to model the relationship between the binary dependent variable (default/no default) characterized by the default indicator process \( D_{i,t} \) with a set of dependent predictor variables. This approach is considered to be the workhorse modelling approach in credit scoring. The rating model estimates for each borrower a 12-month PD which is mapped into a rating class. The rating class is determined by a lower and upper limit which is estimated on quantitative reasoning on the distribution of the estimated PD. The credit rating provides a ranking about the creditworthiness of individual obligors.

The logistic regression allows to model \( P(\text{default} = \text{True}|X) \) by using the logistic function which produces outputs in the interval \([0, 1]\). Hence, the output can be interpreted as probability. In addition, this modelling approach creates marginal effects which are level dependent. \(^1\) (Baesens et al., 2016 pp. 143–150). The PD estimate is in the logistic regression framework given as probability conditional on the realization of a set of independent predictor variables:

\(^1\)This overcomes the problem of using a linear regression approach which does not guarantee PD estimates in the interval \([0, 1]\).
Chapter 2. Theoretical Background

\[ P(\text{default} = \text{True}|X) = \frac{\exp^{X'\beta}}{1 + \exp^{X'\beta}} \]  \hspace{1cm} (2.6)

where \( X \) is a vector of response variables.

As noted by Baesens et al. (2016), these models are typically estimated by using a Maximum-Likelihood approach. In practice, the model for corporate clients typically includes various accounting figures as change in revenue, earnings or debt ratio.

![Figure 2.5: Logistic Function](image)

\[ f(x) = \frac{1}{1 + \exp^{-x}} \]

used in 12-month PD estimation

Figure 2.5 illustrates the logistic function, which is used for the PD estimation with the logistic regression approach. The link function has the needed property to guarantee PD estimates in the interval \([0, 1]\). This is a requirement for any PD estimation and has to hold also in the multi-period context.

One important aspect noted by Baesens et al. (2016, p. 155) is that the ratings estimated with the credit scoring model are typically considered to be “trough-the-cycle” (TTC) because the data used for estimating the model parameters was collected over a longer time horizon (typically it covers a business cycle). Contrary to TTC ratings are “point-in-time” (PIT) ratings. The TTC characteristic of the
produced PD estimates is important when modelling forward looking PD term structure and will be discussed in more detail. In practice the distinction is often challenging and most rating systems are a mixture between TTC and PIT (Gunnvald, 2016, p. 18).

2.2.3 Migration Matrices and PD Term Structure

As indicated by McNeil et al. (2005, pp. 2–5; 339–340), the quantification of credit risk should not be limited to the PD analysis. Furthermore, the concept of migration matrices is important especially when assessing credit risk on a portfolio level. A migration matrix documents rating migrations within a rating system and is therefore an indicator for credit risk. Herbertsson (2018) stresses the wide range of migration matrices not only in credit risk modelling. For example, they can be used for bond pricing (rating based bond pricing). Table 2.1 is a typical example for an empirical migration matrix where the last column indicates the default state. Härdle et al. (2017, pp. 87–90) highlight the possibility to infer the default rate for each rating class directly from the migration matrix. In the case of Table 2.1 the empirical default rate (DR) for rating class AAA is given by the migration \( DR_{AAA} = AAA \rightarrow D = 0 \). Hence, no company starting with a S&P AAA rating at the beginning of 2018 defaulted until the end of 2018. In the context of migration matrices the empirical default rate can be retrieved with the last column, which describes the migration into the default state.

<table>
<thead>
<tr>
<th>From/To</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC/C</th>
<th>D</th>
<th>NR</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>88.89</td>
<td>11.11</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>AA</td>
<td>0.00</td>
<td>92.05</td>
<td>5.50</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>2.45</td>
</tr>
<tr>
<td>A</td>
<td>0.00</td>
<td>0.87</td>
<td>91.97</td>
<td>3.69</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>3.47</td>
</tr>
<tr>
<td>BBB</td>
<td>0.00</td>
<td>0.00</td>
<td>2.81</td>
<td>90.79</td>
<td>1.43</td>
<td>0.00</td>
<td>0.06</td>
<td>0.00</td>
<td>4.91</td>
</tr>
<tr>
<td>BB</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>4.08</td>
<td>83.93</td>
<td>4.15</td>
<td>0.00</td>
<td>0.00</td>
<td>8.84</td>
</tr>
<tr>
<td>B</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>2.80</td>
<td>79.13</td>
<td>3.99</td>
<td>0.98</td>
<td>14.09</td>
</tr>
<tr>
<td>CCC/C</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.51</td>
<td>12.82</td>
<td>46.15</td>
<td>27.18</td>
<td>13.33</td>
</tr>
</tbody>
</table>
Table 2.1 shows typical empirical properties of migration matrices. In most cases ratings remain in their initial state (see the diagonal) even though this property is less pronounced for bad ratings. Also, we observe an increasing default rate which confirms the nature of a credit rating to rank the obligors according to their creditworthiness. In addition, to 12-month migration data empirical rating agencies publish cumulative PD term structure tables. These tables allow to analyze the PD for a longer time horizon for the various rating categories.

<table>
<thead>
<tr>
<th>Rating/Time horizon (Years)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0</td>
<td>0.03</td>
<td>0.13</td>
<td>0.24</td>
<td>0.35</td>
<td>0.7</td>
<td>0.92</td>
</tr>
<tr>
<td>AA</td>
<td>0.02</td>
<td>0.06</td>
<td>0.12</td>
<td>0.22</td>
<td>0.32</td>
<td>0.83</td>
<td>1.04</td>
</tr>
<tr>
<td>A</td>
<td>0.06</td>
<td>0.14</td>
<td>0.23</td>
<td>0.35</td>
<td>0.49</td>
<td>1.28</td>
<td>1.98</td>
</tr>
<tr>
<td>BBB</td>
<td>0.17</td>
<td>0.46</td>
<td>0.8</td>
<td>1.22</td>
<td>1.64</td>
<td>3.44</td>
<td>4.87</td>
</tr>
<tr>
<td>BB</td>
<td>0.65</td>
<td>2.01</td>
<td>3.63</td>
<td>5.25</td>
<td>6.78</td>
<td>12.22</td>
<td>15.17</td>
</tr>
<tr>
<td>B</td>
<td>3.44</td>
<td>7.94</td>
<td>11.86</td>
<td>14.95</td>
<td>17.33</td>
<td>24.21</td>
<td>27.43</td>
</tr>
<tr>
<td>CCC/C</td>
<td>26.89</td>
<td>36.27</td>
<td>41.13</td>
<td>43.94</td>
<td>46.06</td>
<td>50.44</td>
<td>52.8</td>
</tr>
<tr>
<td>Investment grade</td>
<td>0.09</td>
<td>0.25</td>
<td>0.43</td>
<td>0.66</td>
<td>0.9</td>
<td>1.96</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Hull (2010, pp. 292–295) stresses the vact, that empirical cumulative PD curves are monotonically increasing, which can be observed in Figure 2.6. Therefore, the marginal PD is always larger than zero which makes sense as the PD of a rating class never reaches zero. Additionally, one observes that better rating classes have lower cumulative PD curves which is in line with the PD in 12-month migration matrices.

Next, we will present some useful notation in a discrete time setting which will be used for the expected credit loss computation (Eder, 2016, pp. 47–50). The marginal PD $PD(t-1, t)$ can be calculated as:

$$PD(t-1, t) = PD(t) - PD(t-1)$$
Chapter 2. Theoretical Background

Figure 2.6: Empirical PD Term Structure Global Corporate Average Cumulative Default Rates (1981-2018) (Data from Standard & Poor’s RatingsDirect 2018, Table 24)

The cumulative default probabilities is obtained by taking the sum on all $PD(t - 1, t)$ until $t$:

$$PD(0, t) = \sum_{s=1}^{t} PD(s - 1, s)$$

From this equations it is possible to see, how the cumulative PD and the marginal PD are related. Once the PD term structure is estimated, the marginal PD can easily be calculated.

2.2.4 Expected Credit Loss and Loan Pricing

As Sironi and Resti (2007, pp. 251–254) indicate, the concept of expected loss can be used to improve a bank’s performance and risk management. Important
to note is that the concepts presented in this section can be understood in a general way and are not limited to any accounting standard or principle but play a crucial role in the management of financial institutions.

One important application is the area of loan pricing where the expected loss enters into the loan pricing as cost component. If a bank determines the expected loss accurately, the losses in the credit portfolio will be covered by the additional spread. Formally a bank would charge the risk free rate plus an additional spread which equates the expected return of the loan with a risk free investment. Hence

\[(1 + r + d_{EL}) \times [(1 - PD) + (1 - LGD) \times PD] = (1 + r) \quad (2.7)\]

where \(r\) is the risk free rate, \(PD\) is the probability of default, \(LGD\) is loss given default and \(d_{EL}\) is the expected loss. Equation 2.7 can be rewritten as:

\[r + d_{EL} = \frac{r + \frac{PD \times LGD}{1 - PD \times LGD}}{\text{Expected Loss Rate (ELR)}} \quad (2.8)\]

Finally, it is possible to see in equation 2.9 that in this particular case the charged spread is equal to the expected loss.

\[d_{EL} = \frac{\text{ELR} \times (1 + r)}{1 - \text{ELR}} \quad (2.9)\]

This example was included in the thesis because it shows how the calculation of expected credit loss has impact on processes within a bank. This concept is also used in the context of IFRS 9, which will be explained in the next section. Additionally, this example shows the nature of expected loss and its components. As stressed by Sironi and Resti (2007, pp. 251–254), in this context the expected loss is not risk because it is already priced into the loan conditions as additional spread. It is crucial to stress that the risk a bank faces in this context is the deviation of the expected loss while the expected loss is considered to be an additional cost component.
2.3 Credit Risk and IFRS 9

This section provides an introduction to the International Financial Reporting Standard 9 (IFRS 9). As the focus of this thesis is the quantitative PD term structure modelling, the main focus is on the PD parameter requirements and estimation proposed by recent literature.

2.3.1 IFRS 9 in a Nutshell

IFRS 9 is the newly introduced accounting standard for financial instruments, which replaced the previous International Accounting Standard 39 (IAS 39). The standard was published in 2014 by the International Accounting Standards Board (IASB) and became effective as of 01. January 2018. For more detailed references and guidance instructions from the regulator see for example Eder (2016), Basel Committee on Banking Supervision (2015), International Accounting Standards Board (2014a) or International Accounting Standards Board (2014c). Berglund (2016, p. 1) highlights that IFRS 9 aims to overcome the criticism of IAS 39, whose standard for loan provisioning didn’t require banks to provision at a level to cover the actual losses realized in the credit portfolios during the Financial Crisis. The old standard was therefore partly blamed for the existential problems of financial institutions. See International Accounting Standard Board (2001) for the whole IAS 39 framework.

“[…] resolving the weakness identified during the financial crisis that credit loss recognition was too little, too late”

Basel Committee on Banking Supervision (2015, p. 11)

IFRS 9 introduces a paradigm shift for loan provisioning and the loss allowance recognition with the introduction of expected credit loss model (ECLM) for financial instruments in the financial statement. This approach replaces the incurred loss approach under IAS 39. The new approach is expected to provide estimates for the actual losses and to incorporate them already at an early stage into the
bank’s balance sheet. As starting point for the brief introduction we introduce the notation for a financial instrument, whose recognition in the financial statement is the basic requirement for impairment accounting (Berglund, 2016, pp. 1–3).

**Definition 2.3.1. Financial Instrument in IFRS 9.** International Accounting Standard Board (2014, §31.11) defines a financial instrument as “any contract, which gives rise to a financial asset and a financial liability or equity instrument of another entity”. Hence, the definition is very broad and includes typical instruments held by commercial banks as for example cash positions, equity instruments, bonds, loans or financial derivatives (hedging and trading purpose).

Financial instruments have to be recognized on the statement of financial position when “the entity becomes party to the contractual provisions of the instrument” (International Accounting Standards Board, 2014b, §3.1.1).

When the criterion for the initial recognition is fulfilled, financial instruments in IFRS 9 are classified either at amortized cost, fair value through other comprehensive income or fair value through profit or loss. For the classification the so-called business model for the instrument and the cash flow characteristic of the instrument are the determining classification factors (Berglund, 2016, pp. 5–6). International Accounting Standards Board (2014b, §B4.1.1–§B4.1.26) provide guidance how to assess and apply the business models for the classification of financial instruments. The specific requirements for the classification of financial instruments are summarized in Table 2.3, where both criteria are assessed at the same time.

**Business Model:** This criterion is assessed on portfolio basis, for instance, for a portfolio of loans or a equity portfolio. The business model is determined on the intention to collect contractual cash flows or to sell the financial instrument before maturity. Alternatively, it is possible for a business model to both sell financial instruments and collect contractual cash flows. The business model criterion is therefore to verify the intention to trade the instrument (trading intent).
Contractual cash flow characteristics of the financial instrument: This criterion is assessed contrary to the business model criteria on the single instrument level. It verifies, that an instrument is characterized by solely payments of principal and interest on the principal amount outstanding at specified dates.

<table>
<thead>
<tr>
<th>Classification</th>
<th>Business Model</th>
<th>Cash Flow Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amortized cost (AC) (International Accounting Standards Board, 2014b §4.1.2)</td>
<td>Collection of contractual cash flows</td>
<td>Solely payment of principal and interest on the principal amount outstanding at specified dates</td>
</tr>
<tr>
<td>Fair value through other comprehensive income (FVTOCI) (International Accounting Standards Board, 2014b §4.1.2A)</td>
<td>Collection of contractual cash flows and selling financial assets</td>
<td>Solely payment of principal and interest on the principal amount outstanding at specified dates</td>
</tr>
<tr>
<td>Fair value through profit and loss (FVTPL)</td>
<td>Residual category</td>
<td>Residual category</td>
</tr>
</tbody>
</table>

After having defined a financial instrument and the respective classification, the next step is to introduce the impairment model for financial instruments, where we motivate the need for the PD parameter and the estimation of the PD term structure.

### 2.3.2 Impairment and Expected Credit Loss Model

IFRS 9 requires impairment accounting for financial instruments held at amortized cost (AC) or fair value through other comprehensive income (FVTOCI), according to the classification scheme for financial instruments presented in Table 2.3 (International Accounting Standards Board, 2014b §9.5.5.1). In the case of a commercial bank instruments subject to impairment accounting typically cover corporate and retail loans, depending on the bank’s business model also corporate bonds or other instruments would fall in one of the categories. A financial
instrument is credit impaired “when one or more events that have a detrimental impact on the estimated future cash flows of that financial asset have occurred” (International Accounting Standards Board, 2014b, A371–A375). Loss allowance is therefore recognized “for expected credit losses on financial assets […] the accumulated impairment amount for financial assets […] and the provision for expected credit losses on loan commitments and financial guarantee contracts” (International Accounting Standards Board, 2014b, A371–A375). The loss allowance of financial instruments is calculated with the expected credit loss model (ECLM).

The standard defines the basic purpose of the ECL calculation by introducing a neutrality principle for the calculation:

“The purpose of estimating expected credit losses is neither to estimate a worst-case scenario nor to estimate the best-case scenario. Instead, an estimate of expected credit losses shall always reflect the possibility that a credit loss occurs and the possibility that no credit loss occurs even if the most likely outcome is no credit loss.”

International Accounting Standards Board (2014b §B5.5.42)

In the literature and practical application it is common to describe the expected credit loss by the following formula in discrete time, which is very similar to the “classical” definition of the expected loss introduced in Section 2.2. ECL is calculated as “the difference between the cash flows that are due to the bank in accordance with the contractual terms of a financial instrument and the cash flows that the bank expects to receive” (International Accounting Standards Board, 2014b A371–A375).

\[
ECL_i = E_0 \left( \sum_{\tau=1}^{M_i} EAD_{i\tau} \times LGD_{i\tau} \times PD_{i\tau} \times \frac{1}{1 + r_i} \right)
\]  

(2.10)

where \( EAD_{i\tau} \) is an estimate of the exposure at a future default date, \( LGD_{i\tau} \) is an estimate of the loss arising on default, \( PD_{i\tau} \) is an estimate of the likelihood of default over a given time horizon, \( r_i \) is the discount rate used for discounting the
Chapter 2. Theoretical Background

ECL to the corresponding reporting date, \( M_i \) is the exposure maturity (Global Public Policy Committee, 2016, Art. 2.1.2.2).

IFRS 9 explicitly requires to take into account the *time-value of money*. This requirement is met by discounting the future cash-flows to the reporting date (International Accounting Standards Board, 2014b, §B5.5.44). The bank is required to include expected prepayment into the ECL calculation (International Accounting Standards Board, 2014b, §B5.5.51). See for example Global Public Policy Committee (2016) for a more comprehensive summary about the main modelling requirements of the parameters LGD and EAD.

The standard introduces a “bucket approach” for staging allocation, where the defined impairment criteria are summarized as stages respective the migration between the stages. (International Accounting Standards Board, 2014c, p. 16). As a consequence, the stage becomes the main criterion for the calculation of the ECL and the continuous monitoring of the credit risk becomes a key criterion for the staging allocation of financial instruments (Basel Committee on Banking Supervision, 2015, p. 11). Figure 2.7 summarizes the bucket approach for the ECL calculation in IFRS 9 which makes a basic distinction between the calculation of a 12-month ECL and a lifetime ECL.

- **12-month ECL**: This ECL is defined as the portion of the lifetime ECL, when only a period of 12 months is considered for the default event of the financial instrument after the reporting date. In a case, when the expected life of the instrument is shorter than 12 months, the ECL is calculated on the remaining life. The 12-month ECL is required to be weighted by the probability of an occurring default (International Accounting Standards Board, 2014b, §B.5.5).

- **Lifetime ECL**: This ECL is considering the loss which is resulting “from all possible default events over the expected life of the financial instrument” after the corresponding reporting date (International Accounting Standards Board, 2014b, p. 56).

Figure 2.7 summarizes, how financial instruments are allocated to the different stages in IFRS 9 (staging allocation). The stage determines how the ECL is
calculated for the corresponding instruments. Significant changes in the credit risk of the position triggers the migration in a different stage. The estimation approach for the ECL is affected by the migration.

Stage 1 instruments ("Performing"): A financial institution has to recognize a 12-month expected credit loss loss allowance as soon as the financial instrument is purchased or originated according to the recognition definition and if it is not considered as credit impaired (International Accounting Standards Board, 2014b, §5.5.7). The approximation for the expected credit loss has to be revised periodically. International Accounting Standards Board (2014b, §5.5.10) introduces a low credit risk exception. Instruments with low credit risk at the respective reporting date are not required to recognize a lifetime ECL. International Accounting Standards Board (2014b, §B5.5.23) defines low credit risk as for example an investment grade rating from an external rating agency. However, it is possible to use internal measures “if they are consistent with the global understanding of low credit risk”.

Stage 2 instruments (“Underperforming”): Financial instruments allocated to stage 2 have to recognize the lifetime expected credit loss. A financial instrument is allocated to stage 2 if it is not considered to be low credit risk, or if the credit risk has increased in a significant way from the initial purchase or origination credit risk (International Accounting Standards Board, 2014b, §5.5.7). For financial instruments, which are not object to the low credit risk exception, the transition criteria is the increase in credit risk since the initial recognition. In practice the significant increase in credit risk since origination criterion is assessed by using a “SICR-test”. In the case of a negative SICR-test the financial
instrument is allocated to stage 1 otherwise it will be in stage 2. Details on the determination of a significant increase are not in the scope of this work.

**Stage 3 instruments (“Non-performing”):** If a financial instrument is, according to the presented definition, considered to be “credit-impaired”, it has to be allocated to stage 3. In this case loss allowance is equal to the lifetime ECL. The expected credit loss for stage 3 assets will be assessed individually. Contrary to stage 2 the interest revenue is calculated in a different way (Berglund, 2016, pp. 14–15). In the practical calculation of the ECL for stage 3 instruments the PD is set to 1 as the instruments meet the default definition.

The previous two sections have introduced IFRS 9 and especially the impairment requirements for financial instruments. Next, we will motivate the need to model the PD term structure in the IFRS 9 framework and present a practical application in the empirical work.

### 2.3.3 PD Modelling Requirements in IFRS 9

For the conceptual understanding of IFRS 9 PD modelling applications it is important to highlight that IFRS 9 defines a clearer *principle-based* approach than IAS 39. This means that the standard in many cases does not define a concrete methodological framework. It is the individual bank’s task to define a robust methodological framework which is in line with the basic criterion defined by the standard (European Systemic Risk Board, 2017, pp. 3–4). “A bank’s board of directors […] are responsible for ensuring that the bank has adequate credit risk practice […] to consistently determine adequate allowances in accordance with the bank’s stated policies […]” in Basel Comittee on Banking Supervision (2015, p. 1) underlines this aspect. Global Public Policy Committee (2016, Art. 2.3.1.3) summarize the two main PD estimates needed for the ECL calculation in IFRS 9 as follows:

- **12-month PD:** A PD estimate for the default event in the next 12 months or for the remaining lifetime of the financial instrument, if the financial
instruments lifetime is less than a year.

- *Lifetime PD*: A PD estimate for the occurrence of the default event for the remaining lifetime of the financial instrument.

We identify the two most important applications of the PD in the IFRS 9 context in the following two cases.

**PD as key parameter for ECL calculations:** The first and most straightforward application is the use in the calculation of the expected credit loss. The PD is one of the most important parameters needed for the calculation. Though in many aspects principle based, Global Public Policy Committee (2016, Art. 2.3.4.2) notes that the assumption of a constant marginal PD for the determination of the lifetime ECL is not possible in IFRS 9 unless an appropriate analysis would support it. This PD modelling requirement motivates the need for the estimation of a PD term structure.

**PD and SICR-test:** An additional use of the PD lies in the determination of a “*significant increase in credit risk*” because this is a requirement for the migration of a financial instrument into stage 2. Eder (2016, p. 16) notes that in practice often a ratio of PDs or the absolute difference of the PD is assessed for the implementation of the SICR-test. In both cases an estimate for the lifetime PD of the position is needed, which motivates the need for a PD term structure for the aspect of stage allocation in IFRS 9.

Global Public Policy Committee (2016, Art. 2.3.2.2, Art. 2.3.2.5 and Art. 2.3.3.5) summarizes key requirements for a IFRS 9 compliant PD modelling (12-month PD and lifetime PD) as follows:

- PD estimates should be unbiased and not conservative;
- Marginal PD estimates should not be constant over time, though constant marginal PDs are possible if an extensive analysis would support it;
Chapter 2. Theoretical Background

- Every factor, which does not reflect the management's view about the future should be removed (removal of bias towards historical data);

- Default definition should be aligned between IFRS 9 and internal credit scoring model for 12-month PD estimation;

- It is expected to incorporate forward-looking information in the PD estimates (typically through the conditioning of PD estimates on macroeconomic scenarios by using a satellite model);

- PD extrapolation is possible if data is not available for longer time horizons. It is possible to aggregate client segments with similar PD profiles for the PD term structure estimation.

These requirements motivate the PD term structure modelling with extrapolation approaches to comply with all requirements. It is not enough to rely solely on historical data. As pointed out by Eder (2016, p. 45), academia and practitioners have proposed many different PD modelling approaches. The PD is considered to be the most often mentioned risk parameter besides LGD and EAD. The heterogeneity in PD modelling is caused by the potentially large amount of financial instruments, regional differences or differences in the credit portfolio allocation. All modelling approaches have different strengths and weaknesses, and no one works in all cases and under all conditions. Following the classification in Eder (2016, p. 45) the most used PD modelling approaches shall be briefly presented.

- Markov-chain models (matrix models)
  1. Roll rate models
  2. Cohort approach
  3. Duration approach

- Survival models
  1. Vintage models
  2. Non-parametric survival models
  3. Semi-parametric survival models
Chapter 2. Theoretical Background

4. Parametric survival models
5. Mixture cure rate models

- Credit spread models
- Logistic models
- Merton-type models

Eder (2016, p. 45) mentions that the models which can be used depend heavily on the available data and the portfolio characteristics. Credit spread models can only be used with large corporate obligors with quoted debt instruments on the capital market and are therefore not available for private or small corporate portfolios. It is important to keep in mind that “Banks should regularly review their methodology and assumptions to reduce any differences between the estimates and actual credit loss experience” (International Accounting Standards Board, 2014b §B5.5.52).

As noted by Eder (2016, p. 46), current literature suggests Markov-chain models (matrix models) as the most used modelling approaches. This is likely to be the case because of the relatively straightforward implementation into the already available data and IT infrastructure. In this master’s thesis we will focus on a Markov chain based PD modelling approach, which will be presented in more details in the rest of this thesis.
Chapter 3

Markov Chain based PD Term Structure Modelling

This section presents the methodological framework used in the empirical analysis. Markov chain theory with emphasis on applications in finance is motivated by including the main properties and estimation approaches of the input parameters. Furthermore, special emphasis is on the discussion of model assumptions, potential model drawbacks and the quantification of parameter uncertainty. The presented theory is related to the IFRS 9 PD modelling application.

3.1 Markov Chain Theory and PD Modelling

Markov chain theory is a powerful modelling tool and therefore used in many quantitative disciplines, such as physics, engineering, biology, quantitative finance or economics. In this section we use the notation presented in Herbertsson (2018), Lando (2009), Bielecki and Rutkowski (2013) and Pardoux (2008), where all resources are focused on Markov chain theory in quantitative finance.

Herbertsson (2018 pp. 22–25) underlines the long tradition and the various fields of applications for Markov chain theory and Markov processes in finance and in particular in quantitative credit risk modelling applications, which can be
summarized as follows:

- Rating-based term structure models;
- Portfolio credit risk modelling, see for example Bielecki and Rutkowski (2013) for an application;
- Portfolio credit derivatives pricing;
- Markov chain theory can be used to mathematically describe the evolution of credit ratings over time. This application will be studied for the scope of this work in more detail.

Before we can formally define a Markov chain, we have to recall the concept of a stochastic process as presented in Herbertsson (2018, pp. 23–25) or McNeil et al. (2005), where we use the notation for random variables introduced in Section 2.2. See the cited sources for more details on probability theory and other fundamental mathematical “tools” in this context (e.g. conditional expectations, conditional probabilities or filtrations).

**Definition 3.1.1. Stochastic process.** A stochastic process \((X_t)_{t \in T}\) is defined as collection of random variables indexed by \(t \in T\):

\[
(X_t, t \in T) = (X_t(\omega)), t \in T, \omega \in \Omega
\]  

(3.1)

From this definition a Markov chain process can formally be described as it was a class of stochastic process. In this thesis we focus on a discrete time setting.

**Definition 3.1.2. Discrete time Markov chain.** We can define the stochastic process \((X_t)_{t \in T}\) as discrete time Markov chain if:

\[
P[X_t = I_t | X_{t-1} = i_{t-1}, X_{t-2} = i_{t-2}, \ldots, X_0 = i_0] = P[X_t = i_t | X_{t-1} = i_{t-1}]
\]  

(3.2)

holds for any \(t\) and every combination of states \(\{i_0, i_1, \ldots, i_t\}\).
This formal definition defines the Markov property of the process \((X_t)_{t \in T}\) which means that the process “lacks memory” and the expectation is not affected of all information that was generated before \(t - 1\).

A process is defined as time homogeneous if \(P[X_t = j | X_{t-1} = i]\) is not depending on the time factor \(t\) for \(i, j \in E = \{1, 2, \ldots, K\}\). This assumption is important when using real data for the the model calibration. Under this condition, the one-step transition probability between state \(i\) and state \(j\) is given by \(p_{i,j}\) which is defined as:

\[
p_{i,j} = P[X_t = j | X_{t-1} = i]
\] (3.3)

We define \(P\) as the transition matrix for the time homogeneous Markov chain \((X_t)\). \(P\) is defined as \(K \times K\) matrix with \(P_{i,j} = p_{i,j}\) for each pair \(i, j \in E\).

\[
P = \begin{pmatrix} 
    p_{1,1} & p_{1,2} & \cdots & p_{1,K} \\
    p_{2,1} & p_{2,2} & \cdots & p_{2,K} \\
    \vdots & \vdots & \ddots & \vdots \\
    p_{K,1} & p_{K,2} & \cdots & p_{K,K} 
\end{pmatrix}
\]

For every row \(i\) in \(P\) it has to hold that \(\sum_{j=1}^{K} p_{i,j} = 1\), which means that each row sums up to 1.

In this thesis we use Markov chain theory to describe the migration dynamics of credit ratings by obligors in a credit portfolio over time. This allows use to model and extrapolate the credit risk dynamics with a limited amount of data, which makes this application very useful. In the literature this approach is well documented and used in many applications (McNeil et al., 2015, pp. 375–378).

Figure 3.2 visualizes the concept of a time homogeneous Markov process. The probabilities of changing the state remain stable over time and are equal to the initial transition matrix \(P\). Additionally, we note that the state \(D\) in this example is absorbing. This means that the probability of going out this state is zero. Herbertsson (2018, p. 32) notes that this property can be used to describe credit
migrations which are characterized by a default state.

Figure 3.1: Trajectory from Simulated Markov Chain with State Space \( E = \{1, 2, 3\} \)

Figure 3.1 illustrates how a stochastic Markov chain process might evolve over time. For this thesis, Figure 3.1 was created with the R statistical software environment. The source code can be provided upon request. The following \(3 \times 3\) transition matrix with state space \( E = \{1, 2, 3\} \) was used.

\[
P = \begin{pmatrix}
0 & 0.7 & 0.25 & 0.05 \\
0.3 & 0.6 & 0.1 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix}
\]

To better understand the concept of a transition matrix Figure, 3.2 was created with the markovchain package by Spedicato (2017) to visualize the transition matrix for a Markov chain process. For the credit risk application we assume that the state space is characterized by credit ratings.

The matrix is hypothetical and was designed in a way to replicate typical empirical patterns for yearly migration matrices in practice. This example motivates the use of Markov chain theory to describe credit rating migrations in practice. Standard & Poor’s RatingsDirect (2018) reports annual migration matrices,
where one can observe that typically the default rate decreases with the credit rating quality (e.g. better ratings have a lower default rate). Furthermore, it is typical that most ratings remain in their starting rating class. To simulate a Markov chain, we have to add a row to make the matrix a square matrix. The added row is the “default row”, which is typically not available in empirical data. In the case of modelling the evolution of credit ratings the default state indicates the absorbing state. Hence we are assuming that a position in the default state will never recover. In the empirical part we show that this assumption may be not realistic for real data.

Here the Markov property of this particular stochastic process can be explained by the fact that the probability of staying/moving is determined by the initial transition matrix. After each, in this case discrete, time step the probability depends only on the previous state space. In this framework it is possible to determine the future distribution of a transition matrix $P$ by using Theorem 3.1.1. This step is very important as we can derive the future migration probabilities,
which makes this process so useful for a lot of applications.

**Theorem 3.1.1** (Chapman-Kolmogorov theorem). We define $X_t : t = 0, 1, 2, \ldots$ as time homogeneous discrete time Markov chain. $(X_t)_{t \in T}$ is characterized by state space $E = \{1, 2, \ldots, K\}$, transition matrix $P$ and the initial distribution $\alpha$. Under these conditions, for every $t \in \{0, 1, 2, \ldots\}$:

$$p(t) = \alpha P^t$$ (3.4)

and for every $s \in \{0, 1, 2, \ldots\}$:

$$p^{(s)}_{i,j} = (P^s)_{i,j}$$ (3.5)

Hence in a discrete time homogeneous Markov chain the future distribution for any $t$ is fully determined by $P$ and can be calculated by using the mathematical tool of matrix multiplication.

$$P^t = P \cdot P \cdot \ldots \cdot P$$ \text{t times} (3.6)

By using this convenient and useful property, the implementation of a homogeneous Markov chain model for the purpose of IFRS 9 PD term structure modelling is rather straightforward with any mathematical or statistical software package like R, MATLAB or Python. The calculation algorithm uses the estimated transition matrix as input. This matrix is extended by adding a row for the default state, which is typically not present in empirical data. This is needed to guarantee a square $n - \text{by} - n$ matrix with order $n$. See any linear algebra textbook for all properties of a square matrix. An example for that is Meyer (2000). We assume that we have estimate the following transition matrix $\hat{P}$:

$$\hat{P} = \begin{pmatrix}
0.9 & 0.05 & 0.04 & 0.01 \\
0.1 & 0.85 & 0.02 & 0.03 \\
0.05 & 0.1 & 0.8 & 0.05 \\
0 & 0 & 0 & 1
\end{pmatrix}$$
We use the described methodology and obtain the cumulative PD term structure by extracting the last column of $P^t$ for any $t$. After having calculated the cumulative PD term structure, one can easily calculate other properties, such as the marginal PD for each rating class. For the IFRS 9 application we want to extract the marginal PD for each rating class to calculate the lifetime ECL. In Figure 3.3 the resulting PD term structure and the survival rate for the above stated transition matrix are calculated for $t = \{1, 2, \ldots, 15\}$.

**Figure 3.3:** PD Term Structure and Survival Rate in a Markov Chain Model
Figure 3.3 and Figure 3.4 show the properties of the PD term structure when we apply the Markov chain framework. Similarly to the empirical PD term structure in Standard & Poor’s RatingsDirect (2018), the cumulative PDs are monotonically increasing over time. The marginal PDs are not constant. Depending on the rating the marginal PDs are increasing or decreasing, and in the limit they are converging towards one PD. In the next section we will show that this property might not be realistic for real data.

In this introduction to Markov chain theory we have focused our analysis on the time homogeneous discrete case as this will be used in the empirical work. See Herbertsson (2018, pp. 33–40) for the time inhomogeneous case as well as the continuous time setting.

3.2 Markov Chain Models for IFRS 9 PD Modelling

The usage of matrix models with Markov chain models as most prominent example has become the most proposed modelling approach in the academic literature
in the context of IFRS 9. In this section we present a brief literature overview about the use of these models in IFRS 9. Eder (2016) finds in a comprehensive literature review assessing all methodological contributions that matrix models were the most named approach in 2016 for PD modelling. See this source for a systematic presentation of all contributions.

When it comes to the estimation of the transition matrix many contributions name the cohort method as the preferred estimation approach, even though the potential drawbacks of this method are well documented. See for example Conze (2015) or Grünberger (2011). Skoglund and Chen (2016) uses matrix models for the inclusion of the macroeconomic scenarios as required by the standard. The authors stress that matrix models are well suited for a methodological straightforward inclusion of macroeconomic scenarios and the computation of PD conditional on macroeconomic realizations.

Although it is often mentioned, some contributions show potential drawbacks of this modelling approach. Brunel et al. (2015) note, that the use of matrix models in low default rates is potentially connected with unstable and volatile results. These results are connected with the difficulty of estimating a valid transition matrix in such cases because migrations into the default state in good rating classes are scarce. Since there is a limited amount of observations, the relative migration probabilities can change substantially over time, which makes the model unstable. Therefore, the estimation based on the migration matrix on short time periods is critical. In the following article Brunel (2016) confirms the highlighted problems of the previous work. Independently on the IFRS 9 application Lando (2009) stresses extensively the assumption of time homogeneity. The author underlines that results have to be assessed very critical and notes that this assumption is not likely to hold for empirical data.

Eder (2016) summarizes that most contributions stress the biggest advantages of the matrix based approach. These models can be calibrated with a limited amount of internal data and data history. Compared to more sophisticated approaches presented in Section 2.3.3 the data requirements are mostly limited to historical migration matrices, which can be computed from the internal rating system. The obligated use of internal rating systems in many banks guarantees
the availability of clean data for this application. The other modelling approaches require in most cases a lot of very specific historical data to calibrate the models. In many cases it is likely that this type of data has not been gathered and the data recovery would be connected with a fair amount of work and resources.

3.3 Parameter Estimation and Model Validation

In this section we present methods for parameter estimation for Markov chain based PD term structure modelling. Eder (2016, pp. 86–87) notes that aspects of model validation in the context of IFRS 9 are not covered extensively in the literature. This section aims to propose approaches to validate Markov chain models with focus on model assumptions and the stability of the estimated results.

3.3.1 Transition Matrix Estimation

In the practical application of Markov chain theory it is obvious that the entries of the transition matrix have to be estimated from data. In this section we solely focus on the estimation based data coming from a rating system. Important estimation methods are the cohort (discrete time) and the duration (continuous time) method. Other approaches are discussed in the literature but not discussed in this thesis.

The cohort method: The cohort method defines \( t_0, t_1, \ldots, t_n \) as discrete time points in an interval \( t_{k+1} - t_k = \Delta t_k \). Following Christensen et al. (2004) an estimator for each entry in \( M \) over the time interval is given by:

\[
\hat{p}_{ij}(t_k) = \frac{n_{i,j}(\Delta t_k)}{n_i(t_k)}
\]

where \( n_{i,j}(\Delta t_k) \) is the number of firms which have migrated from \( i \) to \( j \) and \( n_i(t_k) \)
are the firms in state $i$ at time $t_k$.

If the Markov chain is assumed to be time homogeneous with available data from $t_0$ until $t_N$, McNeil et al. (2015, p. 378) show that the likelihood is given by:

$$L((p_{jk};(n_{tj}),(n_{tjk})) = \prod_{t=0}^{T-1} \left( \prod_{j=1}^{n} \left( \prod_{k=0}^{n} p_{jk}^{n_{tjk}/n_{tj}} \right) \right)$$  \hspace{1cm} (3.8)

Christensen et al. (2004) show, that the Maximum Likelihood estimator for the stated likelihood is given by:

$$\hat{p}_{ij} = \frac{\sum_{k=0}^{N-1} n_{ij}(\Delta t_k)}{\sum_{k=0}^{N-1} n_i(t_k)}$$ \hspace{1cm} (3.9)

This means that the average of the empirical transition probability estimators is the estimator in the time homogeneous setting. If one removes the assumption of time homogeneity, it is possible to estimate the entries for a period $[t, T]$ with the following estimator:

$$\hat{p}_{ij}(t, T) = \frac{n_{ij}(t, T)}{n_i(t)}$$ \hspace{1cm} (3.10)

As noted by Gunnvald (2016, p. 15), in this case $n_{ij}$ is the number of firms which have migrated from $i$ to $j$ during $[t, T]$. In the time inhomogeneous case the extrapolation and aggregation is not as straightforward as in the time homogeneous case.

**The duration method:** Lando and Skådeberg (2002) show, that the Maximum Likelihood estimator again under the assumption of time homogeneity between $t$ and $T$ is given for $i \neq j$:

$$\hat{p}_{ij}(t, T) = \frac{n_{ij}(t, T)}{\int_t^T Y_i(s) ds}$$ \hspace{1cm} (3.11)

In the duration method the future distribution for any $t$ can be calculated as:
Chapter 3. Markov Chain based PD Term Structure Modelling

\[ M(t) = e^{tG} \]  \hspace{1cm} (3.12)

In this case \( n_{ij}(t, T) \) is the total amount of firms moved from \( i \) to \( j \) during the time interval \([t, T]\). \( Y_i(s) \) is defined as the amount of firms in class \( i \) at the specified time period \( s \).

Gunnvald (2016, p. 16) notes that again the time inhomogeneous case makes it more difficult to extrapolate or aggregate.

In the literature the duration method is considered to be superior to the cohort method as discussed in Lando and Skødeberg (2002). Lando (2009) names as drawback for the cohort method that in the case where the empirical data don’t show a migration event the estimator gives this migration a zero probability. Gunnvald (2016, p. 17) stresses that the duration method allows to use an arbitrary length of estimation window. For the estimation of a 12-month transition matrix in the cohort method one has to use a 12-month estimation window. One potential practical drawback of the duration method is the need to calculate an infinite series expansion to calculate the future distribution which is given by \( e^{tG} \). Even though this is no problem with statistical software, such as R or Matlab, an accurate implementation in Excel is rather difficult. See Gunnvald (2016, p. 17) for mathematical details about the implementation procedure.

3.3.2 Model Validation for IFRS 9 PD Models

As underlined by Baesens et al. (2016, pp. 385–390), model validation is a crucial aspect when implementing a new model within a financial institution. The word validation is derived by the Latin expression validus, which means effective, strong or firm. This process typically takes place after the completion and before brings the model into production. Additionally, a continuous monitoring and validation process is proposed to guarantee a correct model specification. Important to note is the principle that the financial institution is primarily in charge for the validation. The supervisor/auditor reviews the validated model, but does not validate the models themselves. These aspects motivate the inclusion of model
validation in this thesis.

With respect to internal rating systems Basel Committee states that:

“The bank must have a regular cycle of model validation that includes monitoring of model performance and stability; review of model relationships; and testing of model outputs against outcomes.”

Basel Committee on Banking Supervision (2019, Art. 36.33 (6))

Even though this refers to the validation of the internal rating system, in the context of IFRS 9 the implementation of a model validation process is very important. Eder (2016, pp. 86–87) compares various estimation models for IFRS 9. The author notes that the current literature does not cover the aspect of model validation extensively, but is more focused on estimation approaches. In general one distinguishes between quantitative and qualitative model validation.

**Quantitative Model Validation:** In the context of Markov chain models the strong assumptions need to be examined in order to properly discuss and validate the results. We recall that the major methodological assumptions are given by the assumption of time homogeneity and Markov property. An additional important aspect is the estimation of the transition matrix. As presented in the previous section the literature proposes various estimation procedures. It is important to stress the impact of the transition matrix on the resulting PD term structure. Therefore, the stability of the transition matrix shall be analyzed in the context of model validation.

**Qualitative Model Validation:** Following the classification presented in Bae-sens et al. (2016, p. 438) qualitative validation can be subdivided in the following topics:

- Use testing;
- Data quality;
Chapter 3. Markov Chain based PD Term Structure Modelling

- Model design;
- Documentation.

In this thesis the focus is on the quantitative model validation. Therefore, the aspect of qualitative validation is not covered extensively. Nonetheless, validation of input data and data quality are extremely important topics in the practical implementation. It is obvious that without clean data sources it is not possible to bring a new model in production or to maintain it over time. For IFRS 9 PD models the used data is coming from the internal rating model when a Markov chain based approach is used. Baesens et al. (2016, p. 93) note that internal rating models are widely spread in the banking industry for years. Therefore, they should be well calibrated and produce reliable output which can be processed for the IFRS 9 models.

In the next section we will discuss some aspects of quantitative model validation in more detail. We use statistical tools to evaluate the PD term structure estimates in a Markov chain based modelling framework. As the ECL model has a direct impact on a bank’s balance sheet we think that this topic is highly relevant and extends the existing literature.

3.3.3 Analysis of Time Inhomogeneity

Lando (2009) indicate that the assumption of time homogeneity is often made because it allows a rather straightforward estimation of the transition matrix as well as aggregation and extrapolation, which is crucial when it comes to the estimation of the PD term structure. However, numerous studies, such as Trueck and Rachev (2009), Nickell et al. (2000), Haccou et al. (1983) have already in 1983 shown that transition matrices are not constant over time. In the context of finance they depend on various factors, such as industry or the business cycle. This fact can also be observed in the annual data published by the rating agency. See Standard & Poor’s RatingsDirect (2018) for the 2018 report. In this thesis it is therefore important to verify the stability of the credit ratings over time.

The investigation of time inhomogeneity or time stability of credit ratings in
empirical data is in most cases based on the comparison of the average matrix with annual matrices. In this case one might measure the distance between the matrices (norm) as shown in Trueck and Rachev (2009). However, it is not the aim of this thesis to compare methods for analyzing time inhomogeneity. In this thesis we follow the approach presented in Gunnvald (2016) which uses the Matrix $L_2$ norm. This method calculates the cell-by-cell distance between the average matrix and the corresponding annual migration matrices.

In the case of time-inhomogeneity the $L_2$ norm should be zero; however, it is unlikely that zero can be found in empirical data. Therefore, one should expect a “small” $L_2$ norm in the empirical data. A $L_p$ distance measure allows to measure the similarity between series or matrices. A shape based measure compares the values of the data in different manners. The proposed approach builds on the ordinary Euclidian distance measure which is given by:

$$L_p = \sqrt{\sum_{i=0}^{N-1} (x_i - y_i)^2}$$  \hspace{1cm} (3.13)

For the comparison of two matrices $M_1$ and $M_2$ this equation is rewritten as:

$$\Delta M_{L_2}(M_1, M_2) \equiv \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} (M_{1,i,j} - M_{2,i,j})^2}$$  \hspace{1cm} (3.14)

In order to account for the amount of migrations the $L_2$ norm is standardized by $N(N - 1)$ which gives the interpretation of a standard deviation ($\sigma$). This measure is a valid instrument to investigate the time homogeneity of migration matrices over time.

$$\Delta M_{L_2}(M_1, M_2) \equiv \frac{\sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} (M_{1,i,j} - M_{2,i,j})^2}}{N(N - 1)}$$  \hspace{1cm} (3.15)

Trueck and Rachev (2009) use additional to this visual representation of time inhomogeneity a statistical test initially presented in Anderson and Goodman (1957) which is shown to follow a $\chi^2$ distribution. Gunnvald (2016) presents
the analysis of confidence intervals as third way to potentially detect time inhomogeneity. This aspect is briefly presented in the next section. In this thesis we will focus on the calculation of the \( L_2 \) norm as this measure gives a visual representation about the migration matrix over time.

### 3.3.4 Uncertainty Quantification with Confidence Intervals

The PD is a point estimate for the true probability of default. Therefore, the estimation process is connected with estimation risk. The study of the potential variety of the estimate can be done by using confidence intervals, which are able to quantify the potential variation of the estimator. A 99% confidence interval of an estimate is defined as the interval, where in 99% of the times the outcome will be.

In the context of PD estimation in a Markov chain based modelling framework we follow the work of Gunnvald (2016) who uses the Wald confidence intervals for this purpose. The Wald confidence interval is an analytic expression, which requires the assumption that the underlying variable follows a binomial distribution. By computing PD confidence intervals for different years one can use the confidence intervals as additional measure to assess the stability of the PD estimates over time and therefore analyze the assumption of time homogeneity additional to the \( L_2 \) norm.

The estimated PD from an observed sample of independent and identically distributed \( X_i \)'s with sample size \( n \) is given by:

\[
\hat{PD} = \frac{X_1 + X_2 + \cdots + X_n}{n} \tag{3.16}
\]

For “large” \( n \) the Central Limit Theorem (CLT) states, that \( \hat{PD} \) follows a normal distribution.
This expression leads to an analytic expression for the construction of a \((1 - \alpha)\) confidence interval for the PD estimate. The Wald confidence interval is therefore denoted by \(CI_W\) and its analytic expression is given by:

\[
CI_W \pm \kappa \sqrt{\frac{PD(1 - PD)}{n}}
\]  

(3.18)

where \(\kappa\) is the \(1 - \frac{\alpha}{2}\) quantile of the standard normal distribution \(N(0, 1)\). See Gunnvald (2016, p. 5) for a full derivation of the Wald confidence interval.

As noted by Gunnvald (2016, p. 6), an alternative approach would be to use a bootstrap approach for the computation of confidence intervals. In the latter case one simulates migration matrices and computes the corresponding confidence intervals for the PD estimation. Obviously this approach does not allow to use an analytic expression as it is the case in Equation 3.18. This might limit the practical use in firms with limited access/knowledge of more sophisticated statistical software. However, in the case of the empirical work in Gunnvald (2016), a comparison shows that bootstrapped and analytic computation produce very similar results for the confidence intervals. In this thesis we focus the analysis on the computation of the Wald confidence interval for the PD estimates.
Chapter 4

Empirical Analysis

This section provides an implementation of the presented methodological framework with data taken from the Italian banking sector. We implement a time homogeneous Markov chain based PD term structure model in discrete time for the private and corporate sector in Italy. Special emphasis is given on aspects of model validation and the assessment of the correctness of model assumptions. The analysis was implemented in the R statistical software environment (R Core Team, 2018). The programming script can be provided upon request. The section concludes by relating the empirical findings to the IFRS 9 PD modelling requirements.

4.1 Data Description and Descriptive Analysis

For the empirical part of this thesis we have collected data containing yearly credit migration matrices published in the methodological annex of the annual report by Banca d’Italia for the private and the corporate sector in Italy. The data source for the years 2014–2018 is Banca d’Italia (2019) respective Banca d’Italia (2016) for the years 2011–2013. Banca d’Italia provides even older data, and one could theoretically create a longer time series. However, due to the change of the national default definition in 2013, we decided to use all available data within the current default definition to avoid potentially biased results. Banca d’Italia
Chapter 4. Empirical Analysis

(2016) has recalculated the credit migration matrices for the years 2011–2013 with the new default definition, therefore this data is included in the sample. Table 4.1 summarizes the data used for this analysis.

<table>
<thead>
<tr>
<th>Table 4.1: Variable Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>Migration Matrix Private Sector</td>
</tr>
<tr>
<td>Migration Matrix Corporate Sector</td>
</tr>
</tbody>
</table>

Banca d’Italia publishes all yearly credit migration matrix data based on the Italian national information system *Centrale dei Rischi* (CR). Centrale dei Rischi requires Italian banks to report a lot of information regarding the issued credit volume and the credit quality in the bank’s portfolios. Therefore, the database is well maintained and the data quality is very good. Italian banks are required to report to the CR every month all individual positions with an amount larger than 30.000 EUR for performing loans and any amount for non-performing loans. CR aggregates the data and provides the banks with information about the total credit exposure of each client (e.g. a bank is able to see if a client has open loans with other banks in the Italian banking market). The data presented in Table 4.1 contains therefore aggregated data from all Italian banks.  

Banca d’Italia provides data for the private and the corporate sector. According to the Banca d’Italia economic sector classification system, the private sector includes individuals or groups of individuals in their function as consumers and producing families (e.g. individual firms, small firms with a maximum of five employees) in Italy. Individuals in the private sector have income from employment as typical income source. The corporate sector consists of all public and private companies in Italy. 

For the estimation of the yearly migration matrices Banca d’Italia uses the cohort method based on the volume of the total credit exposure reported by the banks to CR. For each reference date, all positions are divided into four classes (*no anomalia*, *scaduti da oltre 90 giorni*, *altri prestiti deteriorati* and *sofferenza*). In

---


---

46
this section we will use the original names for best representation. See Section 2.1 for a description of the default definitions in Italy. Banca d’Italia (2019, pp. 122-123) describes in detail the applied logic for the classification of the exposure. The annual migration matrix was constructed by comparing the population at the beginning of the reference year with the population at the end of the reference year. The migrations are based on the volume of each client against the Italian banking system (clients which have outstanding loans from various banks are aggregated into one single position). Positions that did not have an exposition at the end of the reference date were excluded. Positions for which an actual loss was reported were excluded as well. Additionally to the change of the default definition in 2013, the definition of the classification *altri prestitit deteriorati* was changed in 2015. However, this change has had only a marginal impact and is therefore not considered in the analysis. Table 4.2 contains a migration matrix as retrieved from Banca d’Italia.

<table>
<thead>
<tr>
<th>from/to</th>
<th>No anomalia</th>
<th>Scaduti</th>
<th>Altri prestititi det.</th>
<th>Soff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>No anomalia</td>
<td>626.179</td>
<td>686</td>
<td>11.170</td>
<td>2.869</td>
</tr>
<tr>
<td>Scaduti</td>
<td>214</td>
<td>272</td>
<td>562</td>
<td>183</td>
</tr>
<tr>
<td>Altri prestititi det.</td>
<td>6.006</td>
<td>30</td>
<td>64.785</td>
<td>13.063</td>
</tr>
<tr>
<td>Soff.</td>
<td>316</td>
<td>25</td>
<td>1.392</td>
<td>174.113</td>
</tr>
</tbody>
</table>

In the credit migration matrices Banca d’Italia does not provide a finer distinction for the credit quality in the performing category. This aspect differs to the data published by Standard & Poor’s RatingsDirect (2018) or to data retrieved from a credit scoring model. If this framework was be used in a bank, one would use the credit migration matrices produced by the internal 12-month rating system as input. This difference does not limit this analysis, since the applied framework can be easily generalized to migration matrices of arbitrary size.

The migration matrices provides us with a very dynamic measure about the credit quality in the Italian banking sector from whom various indicators can be calculated. We calculate the non-performing exposure ratio and the non-performing exposure volume for both sectors for the years 2011–2018, which are indicators about the credit quality of the exposure held by Italian banks.
Chapter 4. Empirical Analysis

For both sectors we observe an increasing NPE ratio for the first years, with a reduction in the last years. The level of the NPE ratio is higher for the corporate sector. This indicates a worse credit quality and higher credit risk in the corporate sector. In Italy this fact might be connected with factors like political instability or economic recession in the last years. The volume of the NPE remains stable over time. Overall we find, that Italian banks hold positions with high credit credit risk and large positions of NPE.

4.2 Cure Rate and Danger Rate Estimation

Before the estimation of the PD term structure we present an aspect of migrations from the default state. As already presented in the theoretical introduction,
the current default definition in Italy defines three different default states with
different grades of severity (sofferenze is the worst classification).

During the preliminary work on the data we noted, that migrations out of the
default state in the performing state were happening quite often. We recall that
a Markov chain based PD term structure modelling introduces the absorbing
state assumption, which would imply that a position which enters in the default
state will never recover from it. To evaluate this assumption, we use two special
types of credit migrations, which are often analyzed and used as risk indicator in
practice. These risk indicators are known as danger rate and cure rate (Goracci,
2015).

**Danger Rate:** The danger rate represents the aggregated migration probability
scaduti da oltre 90 giorni and altri prestiti deteriorati → sofferenza. For the
estimation of the danger rate, the original migration matrix is therefore reduced
to three states. The danger rate is an important indicator for the additional credit
risk in the default state, which is not captured when analyzing the aggregated
default state.

**Cure Rate:** The cure rate represents the aggregated migration probability
scaduti da oltre 90 giorni and altri prestiti deteriorati → no anomalia. This
makes the cure rate to an indicator for measuring the likeliness of a recovery
from default back into the status as performing exposure. The cure rate is im-
portant, because it can be used to correct the positions in the default state with
the share which is likely to be still “alive”, but classified as default. In the Markov
chain framework we would expect a cure rate of zero to be in line with the model
assumption.

Table 4.3 shows the data used for the estimation of the empirical cure rate (green)
and danger rate (orange) for the 2017–2018 corporate sector migration matrix.
For the analysis of danger rate and cure rate, the empirical matrix was therefore
“shrunken” to a $3 \times 3$ matrix. For a comparison over time, we have calculated
both indicators for both sectors for the years 2011–2018.
Table 4.3: Cure Rate and Danger Rate Corporate Sector 2017–2018 (Mio. EUR)

<table>
<thead>
<tr>
<th>from/to</th>
<th>No anomalia</th>
<th>Scaduti</th>
<th>Altri prestiti det.</th>
<th>Soff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>No anomalia</td>
<td>626.179</td>
<td>686</td>
<td>11.170</td>
<td>2.869</td>
</tr>
<tr>
<td>Scaduti</td>
<td>214</td>
<td>272</td>
<td>562</td>
<td></td>
</tr>
<tr>
<td>Altri prestiti det.</td>
<td>6.006</td>
<td>30</td>
<td>64.785</td>
<td>13.063</td>
</tr>
<tr>
<td>Soff.</td>
<td>316</td>
<td>25</td>
<td>1.392</td>
<td>174.113</td>
</tr>
</tbody>
</table>

Figure 4.3: Cure Rate Comparison 2011–2018

Figure 4.3 and Figure 4.4 show that both indicators are exposed to a significant variation over time. Additionally, the level is quite different for the corporate and the private sector, with higher rates in the private sector. The danger rate is in both sectors on an overall high level, which implies an additional credit risk factor. The danger rate indicates that the non-performing exposure held by the Italian banks is still exposed to additional credit risk. We suggest that the monitoring of cure rate and danger rate provides very interesting insights in the credit quality of the portfolio.

Having this data at hand, the previously stated absorbing state assumption used in the Markov chain based PD term structure modelling seems not to be realistic.
The estimated cure rate for the private sector exceeds 15% in 2015, but also in the corporate sector the cure rate is on a constant high level. In the next section we elaborate the impact of recovering default for the PD term structure estimation as critical discussion of model assumptions is one of the goals of this work.

4.3 Recovering Defaults in a Markov Chain Framework

We briefly recall from the methodological section that the default state is modelled as absorbing state. This implies that once the firm migrates into the default state, the firm will remain there with a probability of 100% (“the company is dead”). In the previous section we have shown, that in our data this assumption is not fulfilled. This result is mainly because of the default definitions, which typically require to classify a position as default when certain legal criteria defined on national and/or EU level are met. At this point one might ask whether it would
be more appropriate to include the cure rate into the PD modelling process.

As noted by Lando (2009), the absorbing assumption guarantees a monotonic increasing PD term structure. Removing the absorbing assumption would change the interpretation of the PD estimates from an estimation of *having defaulted* at a certain time to *being in default* at a certain time point. Even though the difference seems to be small, only the first interpretation can be used if the goal is to model the PD term structure, as it is the case in the IFRS 9 application.

Gunnvald (2016, p. 29) argues that one can think of ratings as the currently best description of the state of the company. If a firm recovers from the default state, the assessment criteria have changed, as new (especially economic) conditions might have evolved. Therefore, one might think, that the newly evaluated firm is different to the company which was classified as default in the first place. Therefore, it is possible to see defaulted firms as new observations in the sample, which does not contradict the absorbing state assumption. In the next section we will keep the absorbing state assumption for the default state for the PD term structure estimation.

### 4.4 PD Term Structure Estimation

In this section the results from the PD term estimation for the used data is presented. It is important to stress once again, that the applied framework works on arbitrary data coming from any rating system. This aspect makes a Markov chain based approach very useful, since banks typically already use different rating models for various client segments and therefore the data should be already available in most cases.

#### 4.4.1 Cumulative PD Term Structure Estimation

In the methodological section of this work we have shown that the estimation of the transition matrix $\hat{P}$ is a critical step for the PD term structure estimation since it fully determines the future distribution. For the estimation of $\hat{P}$ in the
Chapter 4. Empirical Analysis

time homogeneous case we have used the already presented Maximum Likelihood estimator which is given by:

$$\hat{p}_{ij} = \frac{\sum_{k=0}^{N-1} n_{ij}(\Delta t_k)}{\sum_{k=0}^{N-1} n_i(t_k)}$$

We recall, that this estimator basically calculates the average matrix. Starting from the raw data, we have estimated the average transition matrix for both sectors covering the years 2011–2018. The final transition matrix is represented in percent instead of the credit volume given in the input data.

Table 4.4: Average Migration Matrix Private 2011–2018 (%)

<table>
<thead>
<tr>
<th>from/to</th>
<th>No anomalia</th>
<th>Scaduti</th>
<th>Altri prestititi det.</th>
<th>Soff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>No anomalia</td>
<td>98,293</td>
<td>0,430</td>
<td>1,012</td>
<td>0,266</td>
</tr>
<tr>
<td>Scaduti</td>
<td>24,351</td>
<td>21,379</td>
<td>38,729</td>
<td>15,541</td>
</tr>
<tr>
<td>Altri prestititi det.</td>
<td>8,312</td>
<td>0,669</td>
<td>60,463</td>
<td>30,557</td>
</tr>
<tr>
<td>Soff.</td>
<td>0,495</td>
<td>0,040</td>
<td>0,398</td>
<td>99,067</td>
</tr>
</tbody>
</table>

Table 4.5: Average Migration Matrix Corporate 2011–2018 (%)

<table>
<thead>
<tr>
<th>from/to</th>
<th>No anomalia</th>
<th>Scaduti</th>
<th>Altri prestititi det.</th>
<th>Soff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>No anomalia</td>
<td>94,699</td>
<td>0,520</td>
<td>3,836</td>
<td>0,944</td>
</tr>
<tr>
<td>Scaduti</td>
<td>23,766</td>
<td>11,746</td>
<td>51,357</td>
<td>13,131</td>
</tr>
<tr>
<td>Altri prestititi det.</td>
<td>5,605</td>
<td>0,197</td>
<td>72,874</td>
<td>21,323</td>
</tr>
<tr>
<td>Soff.</td>
<td>0,167</td>
<td>0,009</td>
<td>0,548</td>
<td>99,276</td>
</tr>
</tbody>
</table>

Table 4.4 and Table 4.5 show the estimated transition matrices for the private ($\hat{P}_{Private}$) and the corporate ($\hat{P}_{Corporate}$) sector. For the final PD term structure estimation we have aggregated the three default states in one single default state, as the methodological framework does not consider different default states. Additionally, we have established the absorbing state assumption, which was discussed in the previous section. After the substitution of the empirical cure rate with a zero migration probability, the final transition matrix estimates for both sectors are given by:
For the next step we have implemented the algorithm for the computation of the future distribution of the transition matrix and the extraction of the PD term structure. In this work we have estimated the cumulative PD for \( t = \{1, 2, \ldots, 30\} \). Table 4.6 presents the results for a selected range of \( t \).

\[
\hat{P}_{\text{Private}} = \begin{pmatrix} 0.983 & 0.017 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad \hat{P}_{\text{Corporate}} = \begin{pmatrix} 0.947 & 0.053 \\ 0 & 1 \end{pmatrix}
\]

Table 4.6: Cumulative PD Term Structure Estimates

<table>
<thead>
<tr>
<th>Sector/Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private</td>
<td>0.017</td>
<td>0.034</td>
<td>0.050</td>
<td>0.067</td>
<td>0.083</td>
<td>0.158</td>
<td>0.228</td>
<td>0.291</td>
<td>0.403</td>
</tr>
<tr>
<td>Corporate</td>
<td>0.053</td>
<td>0.103</td>
<td>0.151</td>
<td>0.196</td>
<td>0.238</td>
<td>0.420</td>
<td>0.558</td>
<td>0.664</td>
<td>0.805</td>
</tr>
</tbody>
</table>

We observe in Table 4.6 that the PD curve for the corporate sector is higher, which is obviously because of the higher default rate in the estimated transition matrix. The fact that the credit risk in this sector is higher was already discussed in the previous sections. For \( t = 30 \) we have estimated a cumulative PD of 80.5%, which is a very high value. We see, that the cumulative PD term structure is as expecting monotonically increasing and has a similar shape to the empirical curve presented in Standard & Poor’s RatingsDirect (2018). Figure 4.5 visualizes the results from Table 4.6 and compares the sectors.

As the PD term structure was estimated in discrete time steps from \( \{1, 2, \ldots, 30\} \), in this figure the distance between the estimated data points was interpolated to a smooth function. This shows the functional shape of the PD curve better and allows for a better comparison of the results. We have used this technique also in the following figures.
4.4.2 Marginal PD Calculation

After having estimated the cumulative PD term structure, the marginal PD can be directly calculated by using the already presented formula:

\[ PD(t - 1, t) = PD(t) - PD(t - 1) \]

We use the formula on the estimated PD term structure for \( t = \{1, 2, \ldots, 30\} \). Results for selected \( t \) are reported in Table 4.7.

<table>
<thead>
<tr>
<th>Sector/Year</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private</td>
<td>0.017</td>
<td>0.016</td>
<td>0.016</td>
<td>0.015</td>
<td>0.013</td>
<td>0.012</td>
<td>0.011</td>
<td>0.010</td>
<td></td>
</tr>
<tr>
<td>Corporate</td>
<td>0.050</td>
<td>0.048</td>
<td>0.045</td>
<td>0.043</td>
<td>0.032</td>
<td>0.025</td>
<td>0.019</td>
<td>0.014</td>
<td>0.011</td>
</tr>
</tbody>
</table>

One observes that the marginal PD for the corporate sector is much higher in the first years. However, the marginal PD become more and more similar in the
future, and are almost identical at $t = 30$. This result is not surprising and is explained by the properties of the estimation approach.

In the practical IFRS 9 application the marginal PD is needed for the calculation of the ECL in each year and gives therefore an indication about the impact of the estimated PD term structure on the ECL. In the case of the estimation of a lifetime ECL for a position of the corporate sector we would use the estimated marginal PD to calculate the ECL for each year until the remaining lifetime of the financial instrument. Therefore it is clear, how sensitive the ECL is with respect to the estimated PD term structure.

### 4.5 Model Validation

To appropriately evaluate the stability and meaningfulness of the estimated results, we present an analysis about the model assumption of time homogeneity. This is important in the IFRS 9 context, as a failure of the assumption has implications on the compliance with the regulatory PD requirements in IFRS 9.
4.5.1 Testing for Time Inhomogeneity

In order to evaluate the stability of the migration matrices over time, we have computed the $L_2$ matrix norm for the private and the corporate sector. We have estimated both the $L_2$ norm and a $L_2$ norm which is standardized by the outstanding exposure volume in order to visualize the impact of the credit volume on the $L_2$ estimation.

The average matrix presented in Table 4.4 was used as the reference matrix for the private sector and the average matrix presented in Table 4.5 was used as the reference matrix for the corporate sector. As additional robustness check also the median matrix was computed. The results are very similar to the average matrix as reference matrix and are omitted at this point.

Inspecting Figure 4.7 for the private sector and Figure 4.8 for the corporate sector, we find that the $L_2$ matrix norm in both cases is not constant. The fluctuations are in line with the NPE ratio analysis. We find, that in the last two to three years especially in the corporate sector the overall credit quality has increased, which translates to an increase of the $L_2$ norm. For the private sector this effect is not that clear, because the default rate has been on a significant lower level.

In the case of non-constant migration matrices the used estimator is not the Maximum Likelihood estimator anymore and the predictive power of the estimated PD term structure is likely to be poor. Since for the PD term structure estimation the average matrix was used as transition matrix, the model is not able to account for the current increase in the credit quality properly. In the next section we address this issue.
Figure 4.7: $L_2$ Matrix Norms Private Sector
Chapter 4. Empirical Analysis

Figure 4.8: $L_2$ Matrix Norms Corporate Sector

(a) $L_2$ Matrix Norm 2011–2018

(b) Standardized $L_2$ Matrix Norm 2011–2018
4.5.2 Estimation of Recession and Expansion Matrices

In the previous section it was demonstrated that time inhomogeneity is likely to be present in the used data. From a methodological point of view, it is not straightforward to solve this issue and in the literature a dominating way is not discussed (Gunnvald, 2016, p. 35). Solving this issue with a different modelling approach is not within the scope of this thesis. Therefore we present an approach which might be better suited for IFRS 9 and other purposes in the banking reality.

The average matrix which was estimated in the previous section might be a good solution for the estimation of the “average” state of the economy. In our sample already this assumption is critical, as the sample size is very limited and most of the migration matrices are coming from times with high financial distress. Even though not optimal, the average matrix can be seen as best representation of the average state of the economy. Therefore the results produced by the average matrix might be interpreted as average results. This aspect is problematic in the IFRS 9 context, since the PD modelling requirements require the PD estimate not to be conservative and everything which does not reflect the manager’s view about the future should be removed.

To provide a possible solution and to show the variation of the PD term term structure estimates in this section, the PD term structure was estimated based on the yearly credit migration matrix for each year between 2011 and 2018. Table 4.8 summarizes the results for selected maturities for each year in the sample for the private and corporate sector.

In our opinion, for IFRS 9 purposes the best choice would be to choose the most recent yearly migration matrix as transition matrix. Doing this avoids the problem, that the results are biased towards the historical mean, which is clearly not IFRS 9 compliant. Additionally to the IFRS 9 application one could define migration matrices for various states of the economy and use them for scenario analysis or stress testing application. In this sample we would identify the 2018 migration matrix as the expansion matrix, the average matrix can be used for PD term structure estimates during the average state of the economy and migration matrices in the years 2012–2013 could be used for a recession scenario.
### Table 4.8: Recession and Expansion Matrices Cumulative PD

<table>
<thead>
<tr>
<th>Sector/Year</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
<th>2016</th>
<th>2017</th>
<th>2018</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private5y</td>
<td>0.09</td>
<td>0.10</td>
<td>0.10</td>
<td>0.09</td>
<td>0.09</td>
<td>0.07</td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>Corporate5y</td>
<td>0.22</td>
<td>0.30</td>
<td>0.34</td>
<td>0.31</td>
<td>0.23</td>
<td>0.18</td>
<td>0.14</td>
<td>0.11</td>
</tr>
<tr>
<td>Private10y</td>
<td>0.18</td>
<td>0.20</td>
<td>0.19</td>
<td>0.17</td>
<td>0.17</td>
<td>0.14</td>
<td>0.11</td>
<td>0.09</td>
</tr>
<tr>
<td>Corporate10y</td>
<td>0.39</td>
<td>0.51</td>
<td>0.56</td>
<td>0.52</td>
<td>0.41</td>
<td>0.32</td>
<td>0.27</td>
<td>0.21</td>
</tr>
<tr>
<td>Private15y</td>
<td>0.25</td>
<td>0.28</td>
<td>0.28</td>
<td>0.24</td>
<td>0.25</td>
<td>0.20</td>
<td>0.17</td>
<td>0.13</td>
</tr>
<tr>
<td>Corporate15y</td>
<td>0.52</td>
<td>0.66</td>
<td>0.71</td>
<td>0.67</td>
<td>0.55</td>
<td>0.44</td>
<td>0.37</td>
<td>0.29</td>
</tr>
<tr>
<td>Private20y</td>
<td>0.32</td>
<td>0.36</td>
<td>0.35</td>
<td>0.31</td>
<td>0.32</td>
<td>0.26</td>
<td>0.21</td>
<td>0.17</td>
</tr>
<tr>
<td>Corporate20y</td>
<td>0.63</td>
<td>0.76</td>
<td>0.81</td>
<td>0.77</td>
<td>0.65</td>
<td>0.54</td>
<td>0.46</td>
<td>0.37</td>
</tr>
<tr>
<td>Private25y</td>
<td>0.38</td>
<td>0.42</td>
<td>0.42</td>
<td>0.37</td>
<td>0.38</td>
<td>0.32</td>
<td>0.26</td>
<td>0.21</td>
</tr>
<tr>
<td>Corporate25y</td>
<td>0.71</td>
<td>0.83</td>
<td>0.87</td>
<td>0.84</td>
<td>0.73</td>
<td>0.62</td>
<td>0.54</td>
<td>0.44</td>
</tr>
<tr>
<td>Private30y</td>
<td>0.44</td>
<td>0.48</td>
<td>0.48</td>
<td>0.42</td>
<td>0.44</td>
<td>0.37</td>
<td>0.30</td>
<td>0.25</td>
</tr>
<tr>
<td>Corporate30y</td>
<td>0.77</td>
<td>0.88</td>
<td>0.92</td>
<td>0.89</td>
<td>0.79</td>
<td>0.69</td>
<td>0.61</td>
<td>0.50</td>
</tr>
</tbody>
</table>

### Table 4.9: Recession and Expansion Matrices Marginal PD

<table>
<thead>
<tr>
<th>Sector/Year</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
<th>2016</th>
<th>2017</th>
<th>2018</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private5y</td>
<td>0.018</td>
<td>0.020</td>
<td>0.020</td>
<td>0.017</td>
<td>0.018</td>
<td>0.014</td>
<td>0.011</td>
<td>0.009</td>
</tr>
<tr>
<td>Corporate5y</td>
<td>0.039</td>
<td>0.052</td>
<td>0.057</td>
<td>0.053</td>
<td>0.042</td>
<td>0.033</td>
<td>0.027</td>
<td>0.021</td>
</tr>
<tr>
<td>Private10y</td>
<td>0.016</td>
<td>0.018</td>
<td>0.018</td>
<td>0.015</td>
<td>0.016</td>
<td>0.013</td>
<td>0.011</td>
<td>0.009</td>
</tr>
<tr>
<td>Corporate10y</td>
<td>0.031</td>
<td>0.036</td>
<td>0.038</td>
<td>0.037</td>
<td>0.032</td>
<td>0.027</td>
<td>0.023</td>
<td>0.019</td>
</tr>
<tr>
<td>Private15y</td>
<td>0.015</td>
<td>0.016</td>
<td>0.016</td>
<td>0.014</td>
<td>0.014</td>
<td>0.012</td>
<td>0.010</td>
<td>0.008</td>
</tr>
<tr>
<td>Corporate15y</td>
<td>0.024</td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
<td>0.022</td>
<td>0.020</td>
<td>0.017</td>
</tr>
<tr>
<td>Private20y</td>
<td>0.013</td>
<td>0.014</td>
<td>0.014</td>
<td>0.013</td>
<td>0.013</td>
<td>0.011</td>
<td>0.010</td>
<td>0.008</td>
</tr>
<tr>
<td>Corporate20y</td>
<td>0.019</td>
<td>0.018</td>
<td>0.017</td>
<td>0.018</td>
<td>0.019</td>
<td>0.018</td>
<td>0.017</td>
<td>0.015</td>
</tr>
<tr>
<td>Private25y</td>
<td>0.012</td>
<td>0.013</td>
<td>0.013</td>
<td>0.012</td>
<td>0.012</td>
<td>0.011</td>
<td>0.009</td>
<td>0.008</td>
</tr>
<tr>
<td>Corporate25y</td>
<td>0.015</td>
<td>0.012</td>
<td>0.011</td>
<td>0.012</td>
<td>0.014</td>
<td>0.015</td>
<td>0.015</td>
<td>0.013</td>
</tr>
<tr>
<td>Private30y</td>
<td>0.011</td>
<td>0.012</td>
<td>0.011</td>
<td>0.011</td>
<td>0.011</td>
<td>0.010</td>
<td>0.008</td>
<td>0.007</td>
</tr>
<tr>
<td>Corporate30y</td>
<td>0.012</td>
<td>0.009</td>
<td>0.007</td>
<td>0.008</td>
<td>0.011</td>
<td>0.012</td>
<td>0.012</td>
<td>0.012</td>
</tr>
</tbody>
</table>

61
Chapter 4. Empirical Analysis

Figure 4.9: PD Term Structure Curves 2011–2018

(a) Cumulative PD Term Structure Private Sector

(b) Cumulative PD Term Structure Corporate Sector
Chapter 4. Empirical Analysis

Figure 4.10: Marginal PD Curves 2011–2018

(a) Marginal PD Curves Private Sector

(b) Marginal PD Curves Corporate Sector
4.5.3 PD Confidence Intervals

In this section confidence intervals are used for the 12-month PD estimation using credit migration matrices. The 12-month PD derived from the migration matrix is the starting point of the PD extrapolation executed with the Markov chain approach. The in Section 3.3.4 proposed Wald confidence interval estimator is used for this purpose. For the presented analyses $\alpha = 0.01$, hence the 99% confidence interval is calculated. The calculated results are reported in Table 4.10 and Table 4.11. Figure 4.11 supports the hypothesis of a time inhomogeneity, since the empirical default rates are fluctuating in a significant way over time. We observe that the default rates in the Italian banking sector are decreasing over the last years after peaking in 2011–2012. The credit risk in the corporate sector is higher with overall default rates of almost 8% in 2012 compared to about 2% for the private sector.

The results show that the confidence intervals for the PD estimation are very narrow. The confidence intervals for the private sector are even narrower than in the corporate sector. This is explained by of the lower default rate in the private sector. For this sample the insights from the computation of confidence intervals are limited. The extreme large credit volume and the aggregated positions lead to very narrow confidence intervals. However, for banks with smaller credit portfolios we think that the proposed analysis might be useful.

| Table 4.10: PD Estimates and Confidence Intervals Private Sector |
|-------------------|---|---|---|---|---|---|---|---|
| Year | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
| $\hat{PD}$ | 0.021856 | 0.021381 | 0.018233 | 0.018948 | 0.015153 | 0.011954 | 0.009502 |
| $CI_{low}$ | 0.021856 | 0.021381 | 0.018233 | 0.018947 | 0.015152 | 0.011953 | 0.009502 |
| $CI_{up}$ | 0.021857 | 0.021382 | 0.018234 | 0.018948 | 0.015153 | 0.011954 | 0.009503 |

| Table 4.11: PD Estimates and Confidence Intervals Corporate Sector |
|-------------------|---|---|---|---|---|---|---|---|
| Year | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
| $\hat{PD}$ | 0.068497 | 0.078999 | 0.071172 | 0.051356 | 0.038400 | 0.030503 | 0.022975 |
| $CI_{low}$ | 0.068496 | 0.078998 | 0.071171 | 0.051355 | 0.038400 | 0.030503 | 0.022974 |
| $CI_{up}$ | 0.068498 | 0.079000 | 0.071173 | 0.051356 | 0.038401 | 0.030504 | 0.022975 |
Chapter 4. Empirical Analysis

Figure 4.11: PD Confidence Intervals for Private and Corporate Sector 2011–2018
4.6 Discussion of Results in Context of IFRS 9

The empirical analysis has shown that a Markov chain based PD term structure modelling approach is relatively straightforward to implement. One big advantage is the limited need of historical data for a successful model calibration, which is also mentioned in the literature as important advantage of this approach. However, the transition matrix estimation might become difficult in portfolios with low default rate or in portfolios with a low amount of observations for certain ratings. If the cohort method for the estimation of the transition matrix is used, this might lead to very volatile results. In the case of low default portfolios it could be the case that the empirical default rate for good ratings is almost zero percent. The use of the duration method can overcome the shortcoming of zero defaults. However, the problem with very volatile results remains and is certainly a problem for banks. In the presented analysis both factors were not relevant because the portfolio is on a national level with a very high level of aggregation, and the default rate in both sectors is rather high.

One of the practical benefits of the Markov chain approach is the relative low model complexity. In a bank the model used for IFRS 9 purposes will be assessed by external auditors and should therefore be not too complex from a methodological point of view. This aspect is especially relevant for small and mid-sized banks with limited resources for model development. Additionally, the software requirements are limited. The model can be easily implemented in any mathematical/statistical software package like R, Matlab or Python. An implementation can even be done in MS Excel, which is available in most banks.

For IFRS 9 we suggest, not to use the average matrix for the transition matrix estimation. The PD term structure does not fulfill the criterion of an unbiased and not conservative PD estimate. In our sample the reduction in credit risk in the last years is not fully captured by the average matrix. Therefore, the PD estimates are too conservative with respect to the current default rate level. In the presented analysis the average migration matrix for the corporate sector estimates the cumulative PD for 30 years with 80,5%. Contrary to that, the model calibrated with the 2018 migration matrix estimates a cumulative PD of 50%. Table 4.8 reports the results for the estimation based on the 2018 migration
matrix. These extreme differences are also present for the private sector. We feel that this situation is likely to be present for many banks in Italy who were able to reduce the NPE ratio in their credit portfolio.

The Markov chain based approach is able to produce marginal PD estimates which are not constant and can be used on a portfolio level. Therefore, this regulatory requirement is fulfilled. The presence of a significant cure rate, which is present in the bank's credit portfolios, contradicts the absorbing stage assumption, but is a less severe methodological problem for the PD estimation. The alignment of the used default definition between the internal 12-month PD model and the PD term structure estimation for IFRS 9 is not problematic. A bank should be able to create credit migration matrices from the output of the rating system. In the latter case, the default definitions are aligned.

More problematic is the requirement to remove a potential bias towards historical data. In particular when the average matrix is used for the model calibration, this requirement is certainly not fulfilled. Again using the most recent credit migration matrix for model calibration might be a better approach as in this case the extreme bias towards historical data is partly avoided.

One aspect which was not considered in the analysis is the question whether the migration matrices are produced by a point-in-time (PIT) or through-the-cycle (TTC) estimation. In order to comply with the IFRS 9 requirements, in practice a correction for TTC rating models is made. An additional aspect which is important for IFRS 9 is the inclusion of forward-looking information. In practice, a satellite model is often estimated, which allows to include macroeconomic scenarios into the PD estimates. Although these topics are relevant, they are beyond the scope of this work.

When it comes to the evaluation and comparison of different modelling approaches, Toffano et al. (2017) note that it is important to consider that PD term structure models can not be simply tested by comparing the modelled structure with past observations. This comes from the fact that the regulatory requirements require the PD parameter to be looking forward. Hence, the primary goal is not to fit the model to past data.
Chapter 5

Discussion and a View beyond IFRS 9

In the concluding section of this master’s thesis we discuss and extend the findings from the previous sections. First, a critical discussion about the ECL model is provided. Afterwards, we show how regulators force banks to provide a set of PD estimates for various applications. A limited amount of current literature highlights the necessity to unify these models within a bank to a coherent PD modelling framework. The section concludes by discussing possible suggestions and extensions to this work for future research.

5.1 Expected Credit Loss Mispricing - How Accurate can Expected Credit Loss Modelling be?

As we have already seen, IFRS 9 leaves the bank with relative large flexibility for the concrete implementation of the ECL model. In this context it is important to clarify the risk sources connected with such an approach and to relate them to the concrete case of the PD estimation. The risk in expected credit loss modelling is to not properly evaluating the risk. The expected credit loss itself is considered
to be a cost factor, whereas the actual risk is in the deviation from the estimate. The case of underestimation of the actual risk is a critical case for a bank. We recall from Section 2.2.4 how one could use the expected credit loss criterion for the pricing of loans. Basel Comittee on Banking Supervision (2015) addresses this issue from which we can observe that the ECL is seen as pricing component for the loan pricing within a bank:

“The Committee understands that the rationale for this approach is that the creditworthiness of the counterparty, and thus the ECL upon initial recognition, is taken in account in the pricing of credit at that time.”

Basel Comittee on Banking Supervision (2015, p. 25)

Following Letizia (2017), we can compare the expected credit loss (ECL) with the expected credit premium (ECP) to validate the theoretical coverage of the ECL on a portfolio level. If the ECL on a portfolio level is exactly covered by the expected credit premiums, the fair value of the corresponding credit portfolio is equal to the nominal value of the credits within the portfolio (valuation of the portfolio at par). The following situations are possible in the case of a non-perfect coverage of the credit loss by the credit premiums:

- ECL > ECP: The fair value of the credit portfolio is under par.
- ECL < ECP: The fair value of the credit portfolio is above par.

From the empirical work in this thesis we have learned that the PD modelling in a Markov chain based framework might work well for the first two to three years. Afterwards, the PD estimates are likely to be connected with potentially large estimation errors, where the whole PD term structure depends on the estimation of the transition matrix.

From this rather general observation we derive the following discussion points which challenge the accuracy of expected credit loss and the meaningfulness of
a quantitative staging allocation criterion in a Markov based estimation framework. In this elaboration we solely focus on the PD parameter. Other literature discusses aspects connected with the potential weaknesses of the ECL approach in IFRS 9.

As noted by Eder (2016, pp. 15–16), the lifetime PD is in practice often used for the calibration of the quantitative threshold for a significant increase in credit risk (SICR-test), which triggers a classification in stage 2. As we have observed, especially the meaningfulness of long term PD estimates is critical. This aspect challenges the aspect of staging allocation in IFRS 9 and subsequently the estimation of the ECL.

As noted by Baesens et al. (2016) or Letizia (2017), the concept of expected credit loss works only on a portfolio level with a large number of obligors in that portfolio. In many small and middle-sized banks this assumption is not plausible because they are typically exposed to a rather heterogeneous portfolio. In this context the aspect of default correlation plays a critical role. Herbertsson (2018) notes that default correlation is one of the most important aspects in credit portfolio risk, which is not considered in the IFRS 9 case. We therefore doubt that IFRS 9 is able to overcome all of the “too little, too late” criticism in the case of another shock to the real economy and the financial system, which would again trigger a very significant increase in the default rate the LGD.

An important assumption when using the Markov chain modelling approach is that the migration matrices produced by a rating system are a valid instrument for measuring the actual credit risk for the obligors in the portfolio. However, especially smaller banks with low default rate portfolios are having problems with this assumption. PD estimates produced by any PD term structure modelling approach will be unstable and the PD estimates are potentially biased. All in all, we conclude that one of the key tasks of the risk management department is to monitor and validate the models as well as to contribute to increase the robustness of the PD estimates. This task is challenging since at the same time it is required to comply with all IFRS 9 requirements.

To summarize this elaboration, we conclude that a Markov chain based PD modelling approach is likely to increase the risk of mispricing. Especially the quantifi-
cation of the significant increase in credit risk (SICR-test) is a very critical point and in practice depends heavily on the PD estimates. For stage 2 instruments which require to calculate a lifetime ECL the weaknesses are more pronounced. Therefore, banks with more exposure in stage 2 are more likely to face this risk. In low default rate portfolios the risk of mispricing is less pronounced, but still needs to be continuously monitored.

5.2 Impact of IFRS 9 PD Modelling on Banking Processes

Up to this point of the master’s thesis we have focused the analysis solely on the PD modelling in the IFRS 9 context. As noted by Compagnoni, Elisabetta, et al. (2016), banks face beside the IFRS 9 requirements other regulatory requirements for PD modelling. As stressed by Toffano et al. (2017), PD modelling for different applications has become more important for many banks due to the increasing regulatory requirements. One important aspect is the issue of providing a coherent modelling framework between the different applications. This is important for reasons of model risk, but also under aspects of model development and model maintenance costs. Following Toffano et al. (2017), typical additional application for PD modelling are ICAAP, Recovery Plan or Stress testing.

Internal Capital Adequacy Process (ICAAP): Supervisors require financial institutions to assess their forward-looking capital internal capital adequacy. The risk inventory which has to be considered depends on the individual bank. When a bank is exposed to credit risk in the ICAAP application, the bank is required to provide PD estimates under stressed conditions. See Nouy (2016), Toffano et al. (2017) or European Banking Authority (2016) for elaborations on the ICAAP process with special emphasis on credit risk modelling. In credit risk modelling the most important parameter is again the PD.
Recovery Plan: KPMG International (2017) and European Central Bank (2017) note that additionally to the ICAAP a bank is required to provide to the competent authorities a recovery plan. Contrary to the ICAAP, the Recovery Plan asks banks to develop a range of options to recover from a set of severe scenarios. For the scenarios regarding the deterioration of the credit quality a bank is required to develop adverse forward looking scenarios capturing plausible macroeconomic shocks. The importance of the consideration of credit risk aspects is stressed by the ECB (see for example European Central Bank (2018)).

Stress testing: Already in 1999, Berkowitz (1999) underlines the need for stress testing (input) parameters in credit risk modelling for model validation and stress testing purposes. More recently, Rösch and Scheule (2007) discuss the importance of stress testing in the context of credit risk required by regulators, but also for internal use to further develop the internal models and to allocate capital. These aspects are further discussed by Lopez et al. (2005), or more recently by Bandt et al. (2013). The ability to stress the PD parameter is therefore important for banks and they should be able to compute PDs under adverse scenarios.

In larger banks the complexity of the applications increases fast and makes the models difficult to maintain and to guarantee methodological coherent results among the applications. We follow Toffano et al. (2017) and try to identify the following aspects as the most important ones to guarantee a coherent modelling framework.

Estimation of transition matrices: The estimation of transition matrices are typically not unified across the different applications. Since the estimation of the transition matrix is one of the most important steps, the estimation procedure should be following the same approach and consider different methodological requirements. ICAAP, Recovery Plan and stress testing applications require the migration matrices to be modified. For the credit risk stress scenario the EU-wide banking stress test 2018 requires banks to sterilize the cure rate for stage 3 assets (European Banking Authority, 2018, Art. 1.3.8). A coherent methodological approach should be able to incorporate this requirements.
Inclusion of forward-looking information: Even if this issue was only touched for IFRS 9 in this thesis, the aspect of forward looking information is also relevant for ICAAP, recovery Plan and stress testing (Toffano et al. [2017]). The typical issue is the use of different/not updated macroeconomic econometric models and scenarios for the calculation of the PD estimate conditional on macroeconomic realizations. As noted by Compagnoni, Elisabetta, et al. [2016], the use of coherent scenarios and econometric modelling approaches is important for the integration within different modelling applications.

PD term structure estimation: As noted by Toffano et al. [2017] and Compagnoni, Elisabetta, et al. [2016], ICAAP, Recovery Plan and stress testing applications require to calculate multiperiodal PD estimates under stress and baseline conditions. As already stated, IFRS 9 requires a “best estimate” PD estimate. As consequence bank’s have to model both baseline and stress PD estimates for various time frames which typically start from the estimation of a 12-month PD to 3-year PD for stress testing, whereas in IFRS 9 PD estimates can also cover thirty years.

As we see, there are many different considerations to look at. Because of this complexity and recent importance, the coverage in academic literature is not exhaustive yet. Toffano et al. [2017] provide in a white paper a Markov based modelling framework which aims to include all above describes features and model requirements in a single framework. The main advantage of such an approach for a bank is the fact to align all internal PD models.

5.3 Aspects of Validation and IT Architecture in a Coherent PD Modelling Framework

The operationalization of the identified aspects in Section 5.2 requires a bank to design a coherent and comprehensive IT infrastructure, which is able to combine all aspects into one methodological framework. This is a difficult task, which requires the bank to combine and align a lot of data sources, models, reports.
and documentation. In particular, there is a need to connect the various models starting from the internal 12-month PD model. We suggest that the additional PD models are build upon the 12-month PD model where the approach proposed by Toffano et al. (2017) already provides a sophisticated approach both from a practical and from a rigorous methodological view.

![Figure 5.1: Integrated PD Model Architecture for IFRS, ICAAP, Recovery Plan and Stress testing](image)

In figure 5.1 we designed the model architecture for a integrated modelling framework that is aiming to provide a comprehensive PD modelling approach, which covers all of the regulatory and internal PD requirements. We put special emphasis on the input data, which is essential for the initial model calibration which provides outputs for the subsequent PD term structure calibration.

An additional important aspect is to continuously validate and monitor the PD models, which is especially important. For example, the ICAAP or stress testing applications require the bank to use macroeconomic scenarios. Coherence between the used macroeconomic scenarios is therefore a crucial aspect. For the aspect of model validation we recall the distinction made in Section 3.3, which defines quantitative and qualitative model validation. In the context of PD models we argue that both quantitative and qualitative validation is important. In contrast, especially aspects of data quality and model design are important aspects as the models will be assessed by regulators.

We conclude that aspects of IT architecture and model validation are in practice almost as important as the methodological PD model choice. Therefore, we suggest that all banks should aim for a coherent PD modelling framework with strong emphasis on model validation and an efficient IT architecture. Even though the
implementation of such a framework is connected with large investments, we believe that a bank is able to generate additional value after the implementation. This value comes from less model risk and better calibrated/more robust models, which allow to produce all needed PD calculations.

5.4 Suggestions for Future Research

As already pointed out by Eder (2016, pp. 86–87), literature focuses mainly on the estimation process of various models. Therefore, we feel that the aspect of model robustness and model validation for the proposed models should be investigated further and in more detail. As noted by Toffano et al. (2017), backtesting of PD term structure models is limited. In order to validate the PD term structure against historical data, one would need at least data for a complete credit cycle. Therefore, the need to understand the potential variety of the estimates becomes an important aspect. This suggestion is obviously not only valid for a Markov chain based modelling approach but it is meant for all potential models.

One further suggestion is to investigate dependencies between different PD modelling frameworks within a financial institution to better understand how the different risk parameters are related to each other (correlation effects). Even though the IFRS 9 framework does not consider correlation effects between the various risk parameters, it is still an important aspect. Additionally, the aspect of stress testing/robustness analysis/sensitivity analysis of the modelling input parameters PD, LGD, EAD, migration matrices and macroeconomic scenarios are very important. As indicated by Rösch and Scheule (2007), this aspect is important and the focus of further analysis should be concerned with it.

Finally, we want to add the suggestion that the development of coherent PD modelling approaches are highly relevant and should be investigated further. In particular the inclusion of macroeconomic scenarios for various PD applications is an aspect, which needs to be assessed in more detail. This and the inclusion of various other different PD modelling requirements in one methodological framework is a challenging task, which is interesting for practitioners and academia.
Chapter 6

Conclusion

In this master’s thesis we present a time homogeneous Markov chain based PD term structure modelling approach in an IFRS 9 framework. In IFRS 9, the PD is a critical parameter in the new ECL model for impairment accounting and for the staging allocation procedure. As the impairment model has a direct impact on the loss allowance of financial instruments, the parameter modelling is highly relevant for any bank or financial institution.

We establish a framework to estimate the PD term structure and the corresponding marginal PD for the private and corporate sector of the Italian credit market. Historical migration matrices published by Banca d’Italia are used for the model calibration. We find, that a Markov chain based modelling approach is a very convenient model, which allows to compute the PD term structure with a minimal need of historical data. However, after analyzing the assumption of time homogeneity, we conclude that the predictive power of the estimates is likely to be poor after the three to four years. The analysis of the $L_2$ matrix norm shows, that the migration matrices are not constant over time and the assumption of time homogeneity is critical. The analysis of the 12-month PD and their corresponding confidence intervals calculated from the migration matrices confirms this hypothesis.

We find, that using the average matrix as transition matrix violates IFRS 9 PD modelling requirements in our data. For a more IFRS 9 compliant implemen-
Chapter 6. Conclusion

tation, we suggest to use the most recent yearly migration matrix as transition matrix estimate. This solution is easy to implement and is based on the same methodological framework. For portfolios with low default rates this approach might not be best suited, because the estimation of the transition matrix with the cohort approach can be problematic in cases with no empirical defaults. For rating classes with a low amount of observations, the results are likely to be very volatile when the model is calibrated with new data.

We show the presence of recovering defaults ("cure rate"), which contradicts the absorbing state assumption made for the default state. However, this violation is not too problematic. Lastly, the volatility in the PD term structure is shown by estimating the PD term structure based on yearly migration matrices from 2011–2018 for the private and corporate sector.

This master’s thesis contributes to the existing literature by illustrating the PD modelling process within the IFRS 9 framework with recent data from the Italian banking sector. The empirical results for the Italian credit market show a higher default rate for the corporate sector, which translates in a higher cumulative PD term structure. The calculated marginal PD curves for the corporate sector are higher in the first years, but become more similar for later maturities. Furthermore, this work shows the interdependence between different PD modelling approaches within one financial institution. We propose an integrated approach which allows banks to control all PD models from one framework as we show the impact of IFRS 9 PD modelling on banking processes. The current regulatory environment requires banks to provide PD estimates for various applications. Therefore we think, that coherent PD modelling is becoming more important.

In our opinion, the empirical results underline the importance of model monitoring and model validation for banks. We think, that IFRS 9 partly redefines the role of the risk management. The risk management’s task is not only to monitor the calculated results but also to monitor the model itself to guarantee a coherent and robust implementation. Potential future research topics were already discussed, but we want to stress the importance of model validation for various PD modelling approaches. In this thesis we have focused the analysis one one modelling approach, but in practice there is a large pool of various models in use.
Bibliography


Conze, Antoine (2015). “Probabilities of default for impairment under IFRS 9”. In: *Available at SSRN 2685099*.


Global Public Policy Committee (2016). *The implementation of IFRS 9 impairment requirements by banks*.


International Accounting Standards Board (2014c). *Project Summary*.


Nouy, Danièle (2016). “Supervisory expectations on ICAAP and ILAAP and harmonised information collection on ICAAP and ILAAP”. In: *European Central Bank, Frankfurt am Main*.


Schmidt, Markus (2016). “Lecture Notes in Risk Management”. In: *University of Innsbruck*.

Skoglund, Jimmy and Chen, Wei (2016). “Rating momentum in the macroeconomic stress testing and scenario analysis of credit risk”. In: Available at SSRN 2791524.


