Manipulation in Course Bidding

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ABSTRACT

The Sowi Point System is a bidding mechanism used at the University of Innsbruck allocating course seats among students. The mechanism extents the Standard Bidding Mechanism’s design by incorporating additional aspects as course organization with multiple units, successive bidding rounds and special treatment for exchange students. Research on the basis of Krishna et al. (2004) criticizes the Standard Bidding Mechanism to lack efficiency due to the dual role of bids and promotes the Gale-Shapley Pareto-Dominant Market Mechanism as an alternative mechanism applicable for course allocation that is easily implemented and optimizes the matching outcome.

This thesis investigates the Sowi Point System’s algorithm revealing weak points and opportunities for students to behave strategically. Our theoretical approach emphasises the potential of additional aspects to lengthen the allocation procedure and create opportunities for manipulation, as well. Our analysis supports the findings of Krishna et al. (2004) suggesting that a transfer from the Point System’s algorithm to the Gale-Shapley Mechanism could reduce strategic behaviour and therefore also enhances the course allocation.
1 INTRODUCTION

This thesis examines the Sowi Point System which is a bidding mechanism used for course allocation among students at the University of Innsbruck. Since many studies do not have a standardized course of study, students are able to choose schedules individually. Hence, course enrolment is challenged to coordinate students’ requests over desired courses, obtaining a course allocation each semester.

The University of Innsbruck has different enrolment systems in use. Study programmes belonging to the Faculty of Economics and Statistics as well as the School of Management are organized jointly via the Sowi Point System. The Point System is a bid-based mechanism, resembling an auction. Bidding mechanisms are widely used at business schools endowing students with an artificial currency which they can use to allocate among desired courses during each semester’s enrolment period. Course seats are assigned to the highest bidders and the lowest successful bid of a course, i.e. the lowest bid of a student who is registered in the corresponding course, denoting the course price is market clearing. Therefore, bid-based mechanisms are promoted to create market outcomes. Research refutes bidding mechanisms’ capability to find market outcomes and exposes the efficiency loss resulting from bids’ dual role (Krishna et al., 2004). Students’ submit bids in order to state their claim on courses and to reflect their preferences, as well. When both roles conflicted, as for example more popular courses correspond to relatively high course prices, students have to bid strategically.

The Sowi Point System extents the design of a standard bidding mechanism with several aspects including courses organization with varying units and multiple bidding rounds. We will investigate whether additional aspects facilitate course allocation or create further weak points and opportunities for manipulation.

Therefore, we will clarify the algorithm’s procedure and the design’s weak points. The
thesis is structured as follows: we will provide an overview of the theoretical background in Chapter 2 supporting a basic understanding of allocation problems and desirable properties, characterizing the House Allocation problem and the Housing market. We will cover the concept of a standard bidding mechanism and an alternative, known as the Gale-Shapley Pareto-Dominant Market Mechanism which is promoted by Krishna et al. (2004) to have the potential to enhance a bidding mechanism’s efficiency. In Chapter 3 the Point System will be described in detail outlining distinct aspects which extend the standard bidding mechanism’s design. The Point System’s weak points will be examined in Chapter 4 using properties and the alternative mechanisms presented in Chapter 2 illustrating strategic behaviour. Further, we will propose enhanced outcomes by applying alternative mechanisms lessening manipulation. At last, we will summarize and conclude our findings in Chapter 5.
2 LITERATUR OVERVIEW

The field of market design incorporates the range of allocation problems. Models, outlining a problem’s structure, help to discuss properties and solution concepts. Allocation of course seats among students corresponds to a model assigning indivisible goods to agents, where agents (students) have preferences over objects (courses).

The emphasis of this thesis is the algorithm the Sowi Point System uses. The literature overview helps to embed the problem at hand into the theoretical background and provides information about a mechanism’s desirable properties. Models and mechanisms are outlined concentrating on relevant aspects for argumentation.

2.1 HOUSE ALLOCATION

The house allocation problem is a basic concept for modelling allocations of indivisible goods.

The model consists of a set of agents denoted by $A = \{a_1, a_2, ..., a_n\}$. A set of houses denoted by $H = \{h_1, h_2, ..., h_n\}$. Agents have preferences over houses. A strict preference profile is denoted by $\succ$ = $(\succ_a) a \in A$ such that for each $a \in A$, $\succ_a$ provides a strict preferences relation over $h_i \in H$, $\forall i \in \{1, ..., n\}$. The weak preferences of an agent $a \in A$ is denoted by $\succeq_a$ meaning that for any $h_1, h_j \in H$ with $i, j \in \{1, ..., n\}$, $h_i \succeq_a h_j \iff h_i \succ_a h_j$ or $i = j$.

The outcome of a house allocation problem is called matching. There are no monetary transfers and the number of agents equals the number of houses. Hence, the matching outcome is a one-to-one and onto function $\mu : A \rightarrow H$ where each agent $a \in A$ is assigned to a house $\mu(a)$. Let $\mathcal{M}$ denote the set of matchings.

We will introduce three desirable properties for a mechanism and a matching outcome, respectively, which are applicable for the further analysis of the enrolment problem.
First, a pivotal requirement for the mechanism itself, as well as the matching outcome, is to be efficient. Efficiency is defined in terms of pareto-efficiency. A matching $\mu$ is pareto-efficient if there is no other matching $v$ such that $v(a) \succeq_a \mu(a)$ for all $a \in A$ and $v(a) \succ_a \mu(a)$ for some agent $a \in A$. If the mechanism is finding the pareto-efficient matching outcome, the mechanism itself is pareto-efficient.

Further, a (deterministic direct) mechanism assigns for each house allocation problem a matching. For any $\succ$, let $\phi[\succ] \in M$ denote the matching outcome of $\phi$ for problem $\succ$.

And third, a mechanism is desired be strategy-proof. The mechanism $\phi$ is strategy proof of for any problem $\succ$, agent $a \in A$ and any preference relation $\succ^*_a$:

$$
\phi[\succ_a, \succ_{-a}](a) \succeq_a \phi[\succ^*_a, \succ_{-a}]
$$

(1)

This means when agents reveal their preferences and a central planner assigns a house to each agent using $\phi$ due to their preference profiles, it is a weakly dominant strategy for each agent to report preferences truthfully.

**Serial Dictatorship** (priority mechanism) is a category of mechanism that is applicable to problems as the house allocation as it is strategy-proof as well as pareto-efficient. This mechanism orders all agents randomly or according to a certain priority requirement. A priority ordering is a one-to-one matching and onto function $f \{1, 2, ..., n\} \to A$. So for any $k \in \{1, ..., n\}$, $f(k) \in A$ is the agent with the $k^{th}$ highest priority under $f$. Let $F$ denote the set of orderings, each induces a direct mechanism. The Serial Dictatorship induced by priority ordering $f \in F$ denotes the direct mechanism $\pi^f$, its matching outcome $\pi^f[\succ]$ is found iteratively with the following algorithm:
Serial Dictatorship Algorithm

**Step 1:** Agent with highest priority $f(1)$ is assigned to her top choice.

**Step k:** Agent with $k^{th}$ highest priority $f(k)$ is assigned to her top choice among the remaining objects.

**Termination rule:** All agents are considered.

The mechanism is easily implemented in real-life applications, as for example, a Multi-Categorical Variation is used in Turkey, assigning students to colleges (Balinski and Sönmez, 1999).

### 2.2 HOUSING MARKET

The Housing Market is known as a variation of the House Allocation model. In this setting, each agent is initially endowed with a house, the mechanism aims towards real-allocating houses. The set of agents denoted by $A = \{a_1, a_2, ..., a_n\}$. The set of houses is denoted by $H = \{h_1, h_2, ..., h_n\}$ where each agent $a \in A$ occupies the house $h_a \in H$ with $h_a \neq h_b$ for $a \neq b$. Agents have preferences over houses. A strict preference profile is denoted by $\succ = (\succ_a) a \in A$ such that for each $a \in A$, $\succ_a$ provides a strict preferences relation over $h_i \in H$, $\forall i \in \{1, ..., n\}$. The weak preferences of an agent $a \in A$ is denoted by $\succeq_a$ meaning that for any $h_i, h_j \in H$ with $i, j \in \{1, ..., n\}$, $h_i \succeq_a h_j \iff h_i \succ_a h_j$ or $i = j$.

The properties defined for the house allocation problem concerning a mechanism and a matching also apply to the housing market. Due to the variation of initial structure, another concept is added. The mechanism is desired to be **individually rational**. A matching $\phi$ is individually rational if for each agent $a \in A$, $\phi(a) \succeq_a h_a$. Thus, each agent
is assigned to a house at least as good as her initial endowment. A mechanism finding for each agent such a matching is individually rational itself.

Furthermore, a decentralized solution is introduced which is an exchange economy with indivisible goods. A competitive equilibrium can be reached by decentralized trading. Then, a price vector as a positive real vector assigning a price to each house is needed, i.e. \( p = (p_h)_{h \in H} \in \mathbb{R}^n_+ \) such that \( p_h \) is the price of a house. For a matching a price vector pair \((\phi, p) \in \mathcal{M} \times \mathbb{R}^n_+\) finds a competitive equilibrium for each agent \( a \in A \) if 
\[
p_{\mu(a)} \leq p_{\mu(a)} \text{ (budget constraint) and } \mu(a) \succ_a h \text{ for all } h \in H \text{ such that } p_{\mu(a)} \leq p_{\mu(a)}\text{ (utility maximization).}
\]

The concept of a core is presented as a collection of matchings. Within this collection there is no coalition possible that makes all agents weakly better and one agent strictly better off, even if they trade initial occupied houses among each other. A matching \( \mu \) is in the core if there does no coalition of agents \( B \subseteq A \) exist such that for some matching \( v \in \mathcal{M} \) and for all \( a \in B \), \( v(a) = h_b \) for some \( b \in B \), there is \( v(a) \succ_a \mu(a) \) for all \( a \in B \) and \( v(a) \succ_a \mu(a) \) for some \( a \in B \).

The Gale’s Top Trading Cycle (TTC) is a mechanism applicable to incorporate desired properties.

**Gale’s Top Trading Cycles (TTC) Algorithm**

**Step 1:** Each house points to its owner and agent points to her top choice. Any cycles without intersections are removed and agents are assigned to their choice.

**Step k:** Each remaining house points to its owner and each agent points to her top choice among the remaining. Any cycles without intersections are removed and agents are assigned to their choices.
**Termination rule:** No remaining agents and houses are left.

Both mechanisms introduced, the Random Serial Dictatorship (RSD) as well as the Gale’s Top Trading Cycle (TTC), combine desirable properties which form the basis for further analysis.

### 2.3 COURSE ALLOCATION

The complexity of the course allocation problem exceeds the structures covered by the House Allocation and the Housing Market. The design has to capture that the number of students (agent) and courses (objects) are unequal and that there is a maximum amount of courses students are allowed to take but each student is free to take less courses, too.

One approach treats course allocation as an multi-unit assignment problem which can be solved by the Random Serial Dictatorship as a strategy-proof and ex post pareto-efficient mechanism. Nevertheless, real-life applications of the RSD in course allocation are little. The mechanism is criticized to be intuitively unfair; students ordered at the beginning are “lucky “to have plenty of courses to choose from, whereas students ranked later have to take remaining course (Budish and Cantillon, 2012).

Another approach are bidding mechanisms which are considered as equitably and fair for course allocation. Bidding mechanism constructed comparable to auctions are commonly used at business schools. *Course auctions* function on the basis of an artificial currency. Each student receives a point endowment $E > 0$ for course enrolment. Students allocate their point endowment as bids on desired courses. The algorithm of a Standard Bidding Mechanism ranks all submitted bids of all students for all courses in descending order in one single list. If any course bids of the same amount are submitted a tie-breaking lottery determines the relative ranking between
them. Bids are considered one at a time whether they are successful. A bid is successful only if the corresponding course has a seat remaining and the student’s schedule has an unfilled slot, too.

**Standard Bidding Mechanism**

*Step 1:* Bid that is ranked first is considered to be successful or unsuccessful. Only if bid is successful, student is assigned to corresponding course.

: 

*Step k:* Bid that is ranked at \( k \)th position is considered to be successful or unsuccessful. Only if bid is successful, student is assigned to corresponding course.

**Termination Rule:** All bids in list are considered.

The algorithm attains a course allocation for each student on the basis of her bidding. Course allocation is promoted to be a market outcome which is not supported by corresponding research Krishna et al. (2004). Due to criticism about the lack of direct information about students’ preferences, a derivative bidding algorithm is developed known as the **Gale-Shapley pareto dominant market mechanism**. The authors’ approach is to induce a two-sided matching by incorporating direct information about students’ preferences and applying the deferred acceptance algorithm of Gale and Shapley (Gale and Shapley, 1962). Then, students do not only submit bids but preference rankings over desired courses, as well. Assuming, courses do not have actually preferences over students, the submitted bids are used instead (Krishna et al., 2004):
Gale-Shapley Pareto-Dominant Market Mechanism

**Step 1:** Each student proposes to her most desired courses of the set of all courses she bided for. The number of courses the student proposes to cannot exceed the maximum amount of courses a student is allowed to obtain. Each course keeps students with highest bids on hold. The number of students a course can keep on hold cannot exceed the number of available seats. All other students are rejected.

**Step k:** Each student proposes to her most desired courses of the set of all courses she bided for which have not rejected her in earlier steps. Each course keeps students with highest bids on hold until maximum capacity of course seats is reached. All other students are rejected.

**Termination Rule** When no student is rejected anymore.

Students are assigned to courses which keep them on hold after the algorithm has terminated.

In contrast to the standard bidding mechanism the Gale-Shapley Mechanism has the potential to create more efficient outcomes (Krishna and Ünver, 2008).

The design of the Sowi Point System extents the Standard Bidding Mechanism by several aspects. The following chapter will provide a detailed description of the algorithm and all its characteristics.
3 SOWI POINT SYSTEM

The course enrolment system of the Leopold Franzens University is organized via the LFU online webpage (Sowi Point System). The mechanism in use was developed and programmed in cooperation with the Central Information Service of the University (ZID). The system was applied for the first time in winter semester of 2003/2004.

3.1 COURSES

The registration system as a bidding system with points is used to allocate all course seats for the following studies:

1. Bachelor Program
   a) Economics Sciences – Management and Economics C 033 571

2. Diploma Program
   a) International Economics and Business Studies C1 155

3. Master Program
   a) Business Education C 066 970
   b) Applied Economics C 066 975

4. PhD Program
   a) Social and Economic Sciences C 084 121/140/151/155/170/175
   b) Management C 094 xxx/ C 794 360 xxx

Curricula that contain some social science courses apply to the Point System in those particular cases.
3. **SOWI POINT SYSTEM**

Studies of that matter are:

1. Bachelor Program
   
   a) Commercial Law C 033 500
   
   b) Sports Management C033 626

2. Diploma Program
   
   a) Commercial Law C 115
3.2 THE CONCEPT

Let $S = \{s_1, s_2, ..., s_n\}$ with $j \in \{1, 2, ..., n\}$ denote the set of students. Let $C = \{c_1, c_2, ..., c_k\}$ with $i \in \{1, 2, ..., k\}$ denote the set of courses. Courses are either organized as single courses or course groups. Distinction depends whether there is more than one timeslot for a course $c_i$ available. Timeslots are called units and every course $c_i$ itself is a vector of units, i.e. $\forall c_i \in C \ c_i = \{c_{i1}, ..., c_{ir_i}\}$. If $c_i$ is organized as a single course, there is only one unit available, i.e. $|c_i| = 1$. If course $c_i$ is organized as a course group, there is more than one unit available for course $c_i$, i.e. $|c_i| > 1$.

This structure is equivalent to a **multiple-type housing market** as described in paper of Klaus (2007) with courses being types and units being objects, differing from Klaus in two points:

1. Number of objects $\neq$ number of agents. That is $r_i \neq n \geq 1$

2. There is no initial endowment of units to students

The registration period for courses consists of three successive bidding rounds. Let $T = \{1, 2, 3\}$ denote the different time periods. At the beginning of time period $t \in \{1, 2\}$ each student of the set $S$ receives the same positive bid endowment $E > 0$ via their online account. During each enrolment round, students of $S$ can use the bid endowment to allocate points over desired courses of $C$ as course bids.

$$b_{s_j, c_i, t} \ldots \text{ course bid of student } s_j \in S \text{ on course unit } c_i \in C \text{ at time } t \in T$$

Student $s_j \in S$ places a bid $b_{s_j, c_i, t}$ on as many courses of $C$ as desired. It is in the students responsibility to check for time overlaps of courses. If student $s_j$ wants to be enrolled in a single course, i.e. $|c_i| = 1$, there is only one unit to bid on. Bid $b_{s_j, c_i, t}$
on course $c_i$ indicates the wish for that particular unit. If student $s_j$ wants to be enrolled in a course group, i.e. $|c_i| > 1$, the course has multiple units. Nevertheless, student $s_j$ can only be enrolled in one of the units of $c_i$. Therefore, student $s_j$ places a bid $b_{s_j,c_i,t}$ on the whole course group $c_i$ and has to submit in addition strict preference ranking over units of $c_i$. Only units that are ranked are processed. If units are not ranked the whole bid $b_{s_j,c_i,t}$ will not be processed for that corresponding course group $c_i$.

A ranking of student $s_j \in S$ for every course $c_i$ with $|c_i| > 1$ and $b_{s_j,c_i,t} > 0$ is denoted by $p_{s_j,c_i,t}$. Student $s_j$ indicates no interest in a unit $c_{i_l}$ with $\emptyset$.

$$p_{s_j,c_i,t} = (p_1, \ldots, p_{r_i}) \quad \text{with} \quad p_1, \ldots, p_{r_i} \in \{1, \ldots, r_i\} \cup \{\emptyset\}$$

$$\quad \text{and} \quad p_i \neq p_j \quad \forall i, j \in \{1, \ldots, r_i\} \quad \text{with} \quad p_i \neq \emptyset$$

**Example 1:**

Let a ranking over units for a course be given by $(\emptyset, \emptyset, 1, \emptyset, 3, 2)$.

Then, preferences over units are indicated as $c_{i3} \succ c_{i6} \succ c_{i5} \succ \emptyset$

Shortly put $\forall \quad c_i \in C$

- with $|c_i| > 1 \quad \implies \quad \text{ranking over units must be submitted}$
- $|c_i| = 1 \quad \implies \quad \text{no ranking over units is needed}$

The work load a course contains is measured in ECTS points (European Credit Transfer and Accumulation System). University act states that one academic year amounts to 1500 real-time hours which is equivalent to 60 ECTS points. Consequently, 25 real-time working hours are equal to one ECTS point. Hence, each student is allowed to take courses to the maximum amount of 30 ECTS points per semester. For most of the
above mentioned studies attendance in course $c_i \in C$ equals 7,5 ECTS. So, the number of courses student $s_j$ can take per semester is limited to four. Let $A_{s_j}$ denote the set of courses student $s_j$ attains per semester.

$$A_{s_j} = \{ c_i \in C : \text{student } s_j \text{ is enrolled in course } c_i \} \text{ and } |A_{s_j}| = 4$$
3.3 THE MECHANISM

At the beginning of a bidding round each student of $S$ receives on her online account a bid endowment denoted by $E_{s_j,t}$.

\[ E_{s_j,t} \ldots \text{bid endowment of student } s_j \in S \]

\[ \text{at the time period } t \in T \]

It does not matter if a student is enrolled in more than one study program, each student of the above mentioned studies receives the same bid endowment $E_{s_j,t}$ for an enrolment period in $T = \{1, 2, 3\}$.

Student $s_j$ is allowed to use the point endowment in order to place bids on as many courses of $C$ as desired with the condition that the sum of all submitted bids of $s_j$ is less or equal the point endowment $E_{s_j,t}$.

\[ \sum_{c_i \in C} b_{s_j,c_i,t} \leq E_{s_j,t} \quad \forall \ b_{s_j,c_i,t} \geq 0, \text{ in } t \in T \quad (2) \]

The sum of all bids student $s_j$ has submitted for desired courses of $C$ is denoted by $B_{s_j,t}$.

\[ B_{s_j,t} = \sum_{c_i \in C} b_{s_j,c_i,t} \quad \forall \ b_{s_j,c_i,t} \geq 0, \text{ in } t \in T \quad (3) \]

After closure of each enrolment period $t \in T$, submitted bids are processed.
The Point System’s algorithm processes each enrolment round submitted bids in order to assign students of $S$ to desired courses of $C$. The matching outcome that is found by the algorithm in each enrolment round in $t \in T$ for each student $s_j$ is denoted by $a_{s_j,t}$.

$$a_{s_j,t} \quad \text{course allocation of student } s_j \in S \quad \text{in } t \in T$$

$$a_{s_j,t} = \{c_i \in C : \text{ student } s_j \text{ gets enrolled in course } c_i \in C \text{ in } t\}$$ \hspace{1cm} (4)

Each bidding round is a separate process, the Point System’s algorithm considers in each round submitted bids of the respective round. Therefore, for each bidding round one single list is created which orders all submitted bids. The list that merges all submitted bids in $t$ is denoted by $B_t$.

**Definition 1: Bidding List**

$$B_t = \{b_{s_j,c_i,t} > 0 \mid s_j \in S \text{ and } c_i \in C \quad t \in T\}$$ \hspace{1cm} (5)

The maximum length of $B_t$ is $n \cdot k$, that is every student of $S$ places a positive bid on every available course of $C$.

All bids of $B_t$ are arranged in descending order, i.e. from highest to smallest bid. The highest bid in $B_t$ is denoted by $b_1 = \text{max } B_t$. The smallest bid is denoted by $b_r = \text{min } B_t$.

$$B_t = (b_1, b_2, ..., b_r) \quad \text{s.t. } b_i \geq b_j \quad \text{if } i < j \quad \forall i, j \in \{1, 2, ..., r\}$$ \hspace{1cm} (6)
3. **SOWI POINT SYSTEM**

If order of bids in $B_t$ is not definite, ties are broken with:

1. **Number of Semester**: If two or more bids of the same amount exist a decimal digit equivalent the corresponding student’s semester number is added to each bid $b_p$.

2. **Tie-breaking Lottery**: If ties exist with number of semester, a lottery is used. The lottery adds a random number from the uniform distribution $(0, 0.1)$ to each bid $b_p$.

Result is a definite order of bids in $B_t$, that is $b_p > b_q$ if $p < r \forall p, q \in \{1, 2, ..., r\}$.

All bids that are merged in list $B_t$ are processed separately in descending order.

Enrolment of student $s_j$ in a desired course unit $c_{il}$ requires the corresponding bid $b_p = b_{s_j, c_i, t}$ to be successful. If the following definition holds, the bid $b_p = b_{s_j, c_i, t}$ is successful.

**Definition 2: Successful Bid**

Bid $b_p = b_{s_j, c_i, t}$ denotes the $p^{th}$ bid in $B_t$

- with $p \in \{1, ..., r\}$ and $c^p_i \subset c_i$
- with $c^p_i = \{c_{il} \in c_i : c_{il}$ has a free seat after $b_{p-1}$ is proceeded\}

Bid $b_p = b_{s_j, c_i, t}$ is successful for unit $c^*_{il}$ if

1. student $s_j \in S$ has less than four courses
2. $c^*_{il} \in c^p_i \land c^*_{il} \succ_{s_j} c_{il} \forall c_{il} \in c^p_i$

If bid $b_p = b_{s_j, c_i, t}$ is successful, the student $s_j$ who submitted the bid is enrolled in the
The Point System proceeds each round submitted bids in order to find a course allocation $a_{s_j,t}$ according the following algorithm:

**Sowi Point System’s Algorithm**

**Step 1** List $B_t$ is created.

**Step 2** Procedure starts with $b_1$. If definition of successful bid holds, student $s_j$ is enrolled in course unit $c_{il}$. If definition of successful bid cannot be satisfied, student $s_j$ is not enrolled in course unit $c_{il}$.

**Step 3** Procedure moves to following bid in $B_t$ and processes $b_i$ according description in Step 2.

**Termination Rule** The procedure terminates when $b_r$ of $B_t$ is proceeded.
3.4 FIRST ENROLMENT ROUND

The first enrolment round which is given by $t = 1$ lasts for a time period of approximately two weeks. At the beginning of $t = 1$ every student $s_j$ receives bid endowment $E_{s_j,1}$.

$$E_{s_j,1} = 1000 \quad \text{for} \quad s_j \in S \quad (7)$$

Student $s_j$ can use $E_{s_j,1}$ to place bids on desired courses.

$$b_{s_j,c_i,1} \in \{1, 2, ..., 1000\} \quad \text{bid of student } s_j \in S \text{ on } c_i \in C \quad (8)$$

with the condition

$$\sum_{c_i \in C} b_{s_j,c_i,1} \leq E_{s_j,1} = 1000 \quad \forall \quad b_{s_j,c_i,1} \geq 0 \quad (9)$$

When all students of $S$ have submitted their bids and enrolment period $t = 1$ is closed, bids are processed.

The Sowi Point System’s Algorithm applied in $t = 1$:

**Step 1** List $B_1$ is created.

**Step 2** Procedure starts with the highest submitted $b_1$ of $B_1$. If definition of successful bid holds, student $s_j$ is enrolled in course unit $c_{il}$. If definition of successful bid cannot be satisfied, student $s_j$ is not enrolled in course unit $c_{il}$

**Step 3** Procedure moves to following bid of $B_1$ and processes $b_i$ according description in Step 2.

**Termination Rule** The procedure terminates when $b_r$ of $B_1$ is proceeded.
3. SOWI POINT SYSTEM

For each unit $c_{il}$ of every $c_i \in C$ there is a price denoted by $P_{c_{il},1}$. The **price of a unit** is the lowest successful bid $b^*_i = b^*_{s_j, c_{il}, 1}$ on its course unit $c_{il}$.

$$P_{c_{il},1} = \min_{s_j \in S}\{b^*_{s_j, c_{il}, 1} \in B_1 : \text{student } s_j \text{ gets enrolled in unit } c_{il} \in c_i\} \quad (10)$$

Consequently, all bids that are successful for course unit $c_{il}$ of $c_i \in C$ are greater or equal its price: $b^*_i \geq P_{c_{il},1}$ with $b_i \in B_1$. Vice versa, all bids that are unsuccessful for course unit $c_{il}$ of $c_i \in C$ are smaller than the course price $p$: $b_i < P_{c_{il},1}$ with $b_i \in B_1$.

The sum of all bids student $s_j \in S$ has submitted in the first enrolment round that are successful is denoted by $B^*_{s_j,1}$.

$$\sum_{c_i \in C} b^*_{s_j, c_i, 1} = B^*_{s_j,1} \quad \forall \quad b^*_{s_j, c_i, 1} \geq 0 \quad (11)$$

Vice versa, the sum of all bids student $s_j \in S$ has submitted in $t = 1$ that are unsuccessful is denoted by $U_{s_j,1}$.

$$\sum_{c_i \in C} b_{s_j, c_i, 1} = U_{s_j,1} \quad \forall \quad b_{s_j, c_i, 1} \geq 0 \quad (12)$$

The sum denoted by $B_{s_j,t}$ comprises all successful as well as unsuccessful bids for student $s_j \in S$ in $t = 1$.

$$B_{s_j,1} = B^*_{s_j,1} + U_{s_j,1} \quad (13)$$

There is the possibility for students to drop an assigned course. Therefore, student $s_j$ has to deregister online before the second enrolment round starts. The bid $b^*_{s_j, c_{il}, 1}$ for that corresponding course $c_i$ falls into the sum of student $s_j$’s unsuccessful bids $U_{s_j,1}$. 
There is a transfer of points from the first to the second enrolment round for student $s_j \in S$ denoted by $T_{s_j}$. Points that are transferred from the first to the second bidding period are added up to the bid endowment student $s_j$ receives for period $t = 2$. So in the second enrolment round, student $s_j$ can make use of these points again. Points of student $s_j$ that are transferred are

1. Unused points

2. The amount of points of all unsuccessful bids $U_{s_j,1}$

Only the sum $B_{s_j,1}^*$ of student $s_j$’s successful bids are subtracted from her bid endowment $E_{s_j,1}$. Therefore, the transfer $T_{s_j}$ is given by

$$T_{s_j} = E_{s_j,1} - B_{s_j,1}^* \quad \forall \quad s_j \in S \quad (14)$$

For each student of $S$ a course allocation is implemented when enrolment round $t = 1$ is finished and all bids are processed. Given by

$$a_{s_j,1} = \{c_i \in C : b_{s_j,c_i,1} \text{ was successful}\} \quad (15)$$
3.5 SECOND ENROLMENT ROUND

The second enrolment round is denoted by \( t = 2 \). For bidding in the second enrolment round the initial set of \( C \) gets updated. Courses that have no free seats left are excluded. Let \( C_2 \) denote the set of courses available in \( t = 2 \).

\[
C \supseteq C_2
\]

(16)

The set \( C_2 \) is still organized in single courses and course groups.

At the beginning of \( t = 2 \), each student \( s_j \in S \) receives a point endowment \( E_{s_j,2} \). Students who already got enrolled in four courses of \( C \) cannot make use of the endowment, i.e. can not register any bids in \( t = 2 \).

\[
E_{s_j,2} = 1000 \text{ for } s_j \in S
\]

(17)

In addition to the bid endowment for the second bidding round, student \( s_j \) gets her transfer \( T_{s_j} \) of points from \( t = 1 \). Thus, the amount of points student \( s_j \) has available for bidding in the second enrolment round is depending on the transfer \( T_{s_j} \). A bid in \( t = 2 \) is denoted by \( b_{s_j, c_i, 2} \). Let \( B_{s_j, 2} \) denote the sum of all bids student \( s_j \) submits in \( t = 2 \). Student \( s_j \) can place bids on as many courses of \( C_2 \) as desired. Student \( s_j \) has to check for time overlaps of courses individually. The sum of all bids student \( s_j \) places in \( t = 2 \) is bound to the sum of bid endowment \( E_{s_j,2} \) and transfer \( T_{s_j} \).

\[
\sum_{c_i \in C} b_{s_j, c_i, 2} = B_{s_j, 2} \quad \forall b_{s_j, c_i, 2} > 0
\]

(18)

\[
B_{s_j, 2} \leq E_{s_j,2} + T_{s_j} \quad s_j \in S
\]

(19)
The procedure of bidding in $t = 2$ is similar to $t = 1$.

Thus, the **Sowi Point System’s Algorithm** applied in $t = 2$:

**Step 1** List $B_2$ is created.

**Step 2** Procedure starts with $b_1$ of $B_2$. If definition of successful bid holds, student $s_j$ is enrolled in course unit $c_{il}$. If definition of successful bid cannot be satisfied, student $s_j$ is not enrolled in course unit $c_{il}$

**Step 3** Procedure moves to following bid of $B_2$ and processes $b_i$ according description in Step 2.

**Termination Rule** The procedure terminates when bid $b_r$ of $B_2$ is proceeded.

In $t = 2$ the price of each unit $c_{il}$ of every $c_i \in C_2$ is determined, i.e. $P_{c_{il}, 2}$.

\[
P_{c_{il}, 2} = \min_{s_j \in S} \{ b_{sj, c_{il}, 2} \in B_2 : \text{students gets enrolled in unit } c_{il} \in c_i \}\]  

(20)

There is no transfer of points from $t = 2$ to $t = 3$ anymore. the second to the third enrolment round. Unused points expire.

Declarations of course prices in $t = 2$ are only used in the **bidding history** which is supposed to give students some orientation. Therefore information about students’ previous bidding behaviour in the first and second enrolment of the past two semesters is provided. The history is organized course-wise structured according to the division into single courses and groups. There is for each course unit $c_{il} \in C$ information about course number and available seats. In addition, for each $c_{il} \in C$ the highest successful bid $b^*_i$, the mean of successful bids, and the lowest successful bid, i.e. the price, is documented.
Result of bidding round $t = 2$ is course allocation $a_{s_j, 2}$:

$$a_{s_j, 2} = \{ c_i \in C_2 : b_{s_j, c_i, 2} \text{ was successful} \}$$  \hspace{1cm} (21)
3.6 THIRD ENROLMENT ROUND

The third enrolment round is denoted by \( t = 3 \). The third round takes place when courses have already started and course instructors had the chance to update course lists. It is mandatory for each student of \( S \) who got enrolled in course unit \( c_i \) of \( c_i \in C \) to confirm enrolment in person at the first course unit meeting. For enrolment round \( t = 3 \) students can bid upon course units from the set \( C_3 \).

\[
C \supseteq C_2 \supseteq C_3
\]  

(22)

In comparison to the endowments for the previous rounds, in \( t = 3 \) there is no real point endowment that can be depleted. Students of \( S \) have only one point available. This point can be placed multiple times on desired units of \( C_3 \). The bid \( b_{s_j,c_i,3} = 1 \) has rather a symbolic meaning by indicating the student \( s_j \)’s interest in enrolment. The point available has no numerical value.

\[
b_{s_j,c_i,3} \in \{1, \emptyset\} \quad \forall \ b_{s_j,c_i,3}
\]  

(23)

If a course is organized as a course group with \( |c_i| > 1 \) in \( t = 3 \), it is necessary for student \( s_j \) to submit a ranking over desired course units.

The main difference to the previous bidding rounds is the ordering of submitted bids in \( t = 3 \). Since all bids in \( t = 3 \) are of the same amount, i.e. one point, bids have to be ordered creating \( B_3 \). There is no advantage for students in a higher semester anymore, i.e. there is no number of semester. That means \( B_3 \) is created only on the basis of a tie-breaking lottery. A random number from the uniform distribution \((0, 1)\) is added to each bid \( b_{s_j,c_i,3} \). Resulting in a definite order of bids in \( B_3 \).
The **Sowi Point System**’s Algorithm applied in \( t = 3 \):

**Step 1** List \( B_3 \) is created.

**Step 2** Procedure starts with \( b_1 \) of \( B_3 \). If definition of successful bid holds, student \( s_j \) is enrolled in course unit \( c_{dl} \). If definition of successful bid cannot be satisfied, student \( s_j \) is not enrolled in course unit \( c_{dl} \).

**Step 3** Procedure moves to following bid of \( B_3 \) and processes \( b_i \) according description in Step 2.

**Termination Rule** The procedure terminates when bid \( b_r \) of \( B_3 \) is proceeded.

Since all courses are allocated randomly upon students in \( t = 3 \), there are neither prices for course units \( c_{dl} \) of \( c_i \in C \) nor a bidding history published.

The course allocation for student \( s_j \in S \) in \( t = 3 \) is denoted by

\[
a_{s_j,3} = \{c_i \in C : b_{s_j,c_i,3} \text{ was successful}\}
\]
3.7 HÄRTEFALLVERGABE

Students are allowed to enrol in up to four courses each semester. Studies’ schedules require four course completions each semester in order to graduate whilst minimum duration of study. All students of $S$ who are not enrolled in up to four courses at the end of the three bidding rounds $T = \{1, 2, 3\}$ have one last opportunity, called the Härtefallvergabe, to register. Therefore, students can submit a hand-written request for a desired course. It is neither necessary that the requested course has a seat remaining nor is the student required to bid on the requested course during a previous bidding round of $T$.

The dean of studies handles all requests. If a student requests a seat in a course with free capacity, the student is added to the corresponding course. If a student requests a course without free capacity, the dean is allowed to add another seat to a filled up course.
4 WEAK POINTS

We have described in the previous Chapter 3 the Point System’s structures including course organization with multiple units and successive bidding rounds. The Point System extents the standard bidding mechanism’s design by incorporating several additional aspects. In this chapter, we will focus on some of the aspects. We analyse their impacts on the matching outcome and expose possibilities for strategic behaviour. We will contrast the Point System’s outcome with the alternative mechanisms that we have introduced in Chapter 2 showing their potential to solve for weak points and to enhance the allocation procedure.

Dual role of bids

Bids are essential for a bidding mechanism’s procedure, research on the basis of Krishna et al. (2004) proposes the direct creation of inefficiency due to their dual role. Challenging the function of bids provides the starting point for our discussion of weak points. As described, students are allowed to bid on as many courses as desired and allocate their point endowment freely during bidding periods. The algorithm proceeds submitted bids in descending order assigning course seats to the highest bidders. Thus, submitting a high bid corresponds to a student’s strong desire to be enrolled in that particular course. In this way, direct information is given about the claim a student has on a course. In addition, a student’s bidding behaviour implicitly reflects preferences over courses. The dual role of bids conflict if bidding behaviour is not consistent with true preferences. A source for ambiguous bidding behaviour and inefficient point allocation is that courses which are more popular than others require students to bid relatively high amounts of points to have a chance to be subscribed. Bidding a high amount of points on a less popular course might reflect true preferences but is a waste of points, vice versa. This
conflict constitutes a direct source of strategic behaviour and inefficiencies. The following example will illustrate the conflict and its consequences.

Example 2:
Suppose for student $s_1$ the set of desired courses is denoted by $C = \{c_1, c_2, c_3\}$. She has the capacity to be enrolled in up to two courses and each course of $C$ has one free seat left. Student $s_1$’s preferences over course enrolment are: $c_1 \succ c_2 \succ c_3$. We assume that student $s_1$ has a point endowment of $E_{s_1} = 11$ available and the bidding history of the previous year gives the following course prices:

<table>
<thead>
<tr>
<th>Courses</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Course price</td>
<td>1</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

We assume that student $s_1$ forms beliefs correctly on the basis of the bidding history meaning allocating points as suggested by the history makes each bid high enough to be market clearing for the corresponding course. That gives the following bidding behaviour:

$$b_{s_1,c_1} = 1 \quad b_{s_1,c_2} = 5 \quad b_{s_1,c_1} = 5$$

If beliefs are formed correctly so that $b_{s_1,c_1} = 1$ has the potential to be market clearing, it is a waste of points for student $s_1$ under a market approach to bid more than one point for course $c_1$. However, with this bidding behaviour and without direct information about preferences, the algorithm will assign student $s_1$ to courses $c_2$ and $c_3$. As bids are processed in descending order $b_{s_1,c_2}$ and $b_{s_1,c_3}$ will be considered first. If student $s_1$ wants to be enrolled in course $c_1$ she has to allocate points strategically.

This example illustrates the discrepancy that derives from the dual role of bids. Bids as the crucial instrument of a bidding mechanism are a fundamental source of inefficiencies and strategic behaviour. Therefore, the dual role of bids constitutes the first weak point of the standard bidding mechanism as well as the Sowi Point System.

The research of Krishner et al. (2004) focusses on this critical point. The authors present
the **Gale-Shapley Pareto-Dominant Market Mechanism** (c.f. Chapter 2.3) as an alternative mechanism that has the potential to improve the outcome without wasting points by incorporating additional information about students preferences.

The following example compares the **Gale-Shapley Mechanism** procedure and outcome with the **Point System**.

**Example 3:**

Suppose the set of students is $S = \{s_1, s_2, s_3, s_4, s_5\}$ and the set of courses is $C = \{c_1, c_2, c_3, c_4\}$. Courses are organized as single courses, there are no units $|c_i| = 1$. The list of available seats is $a = (2, 3, 3, 2)$. Students can be enrolled in up to two courses each $|A_{s_j}| = 2$. We assume that courses $c_1$ and $c_2$ are more popular. Bid endowment for each student is $E_{s_j} = 100$. Bid matrix of each student is denoted by $b_{s_j}$ and preference ranking by $p_{s_j}$.

**Student 1** ($s_1$): $b_{s_1} = (52, 40, 5, 3)$ and $p_{s_1} : c_1 \succ c_2 \succ c_3 \succ c_4 \succ \emptyset$

**Student 2** ($s_2$): $b_{s_2} = (50, 8, 7, 35)$ and $p_{s_2} : c_1 \succ c_4 \succ c_3 \succ c_2 \succ \emptyset$

**Student 3** ($s_3$): $b_{s_3} = (30, 16, 15, 39)$ and $p_{s_3} : c_2 \succ c_3 \succ c_4 \succ c_1 \succ \emptyset$

**Student 4** ($s_4$): $b_{s_4} = (20, 38, 19, 23)$ and $p_{s_4} : c_3 \succ c_1 \succ c_4 \succ c_2 \succ \emptyset$

**Student 5** ($s_5$): $b_{s_5} = (24, 46, 1, 29)$ and $p_{s_5} : c_4 \succ c_2 \succ c_3 \succ c_1 \succ \emptyset$

The algorithm of the **Sowi Point System** ranks all submitted bids in descending order in $B_1$ and processes them successively assigning course seats to the highest bidders (c.f. Chapter 3.2). The following table shows the ordering in $B_1$ and marks successful bids:
4. WEAK POINTS

<table>
<thead>
<tr>
<th>Points</th>
<th>52</th>
<th>50</th>
<th>46</th>
<th>40</th>
<th>39</th>
<th>38</th>
<th>35</th>
<th>30</th>
<th>29</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Course</td>
<td>(c_1)</td>
<td>(c_1)</td>
<td>(c_2)</td>
<td>(c_2)</td>
<td>(c_1)</td>
<td>(c_1)</td>
<td>(c_2)</td>
<td>(c_2)</td>
<td>(c_1)</td>
<td>(c_4)</td>
</tr>
<tr>
<td>Student</td>
<td>(s_1)</td>
<td>(s_2)</td>
<td>(s_5)</td>
<td>(s_1)</td>
<td>(s_3)</td>
<td>(s_4)</td>
<td>(s_2)</td>
<td>(s_3)</td>
<td>(s_5)</td>
<td>(s_5)</td>
</tr>
</tbody>
</table>

succ. succ. succ. succ. succ. succ.

<table>
<thead>
<tr>
<th>Points</th>
<th>23</th>
<th>20</th>
<th>19</th>
<th>16</th>
<th>15</th>
<th>8</th>
<th>7</th>
<th>5</th>
<th>3</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Course</td>
<td>(c_2)</td>
<td>(c_3)</td>
<td>(c_4)</td>
<td>(c_3)</td>
<td>(c_4)</td>
<td>(c_3)</td>
<td>(c_4)</td>
<td>(c_3)</td>
<td>(c_4)</td>
<td>(c_3)</td>
</tr>
<tr>
<td>Student</td>
<td>(s_4)</td>
<td>(s_4)</td>
<td>(s_4)</td>
<td>(s_3)</td>
<td>(s_3)</td>
<td>(s_2)</td>
<td>(s_2)</td>
<td>(s_1)</td>
<td>(s_1)</td>
<td>(s_5)</td>
</tr>
</tbody>
</table>

succ. succ. succ.

The following course allocation is obtained:

\[
\begin{align*}
a_{s_1}^{PS} & a_{s_2}^{PS} a_{s_3}^{PS} a_{s_4}^{PS} a_{s_5}^{PS} \\
& (c_1, c_2) (c_1, c_2) (c_3) (c_3, c_4) (c_2, c_4)
\end{align*}
\]

Course prices that are realised:

\[
P_{c_1}^{PS} = 50, \quad P_{c_2}^{PS} = 35, \quad P_{c_3}^{PS} = 16, \quad P_{c_4}^{PS} = 19
\]

In contrast the algorithm of the **Gale-Shapley Mechanism** takes not only students’ bids but also preference rankings into account (c.f. Chapter 2.3):

**Step 1:**

- Student \(s_1\) points to \(\{c_1, c_2\}\)  Student \(s_2\) points to \(\{c_1, c_4\}\)
- Student \(s_3\) points to \(\{c_2, c_3\}\)  Student \(s_4\) points to \(\{c_3, c_1\}\)
- Student \(s_5\) points to \(\{c_4, c_2\}\)
Courses $c_2$, $c_3$, $c_4$ and $c_5$ do not exceed their capacity proposing students are kept on hold. Course $c_1$ has three offers: $b_{s_1,c_1} = 52$, $b_{s_2,c_1} = 50$ and $b_{s_4,c_1} = 38$. Student $s_4$ is rejected, the other students are kept on hold.

**Step 2:**

Student $s_1$ points to $\{c_1, c_2\}$, Student $s_2$ points to $\{c_1, c_4\}$, Student $s_3$ points to $\{c_2, c_3\}$, Student $s_4$ points to $\{c_3, c_4\}$, Student $s_5$ points to $\{c_4, c_2\}$

Courses $c_1$, $c_2$, $c_3$ and $c_5$ do not exceed their capacity proposing students are kept on hold. Course $c_4$ has three offers: $b_{s_2,c_4} = 8$, $b_{s_4,c_4} = 19$ and $b_{s_5,c_4} = 24$. Student $s_2$ is rejected, the other students are kept on hold.

**Step 3:**

Student $s_1$ points to $\{c_1, c_2\}$, Student $s_2$ points to $\{c_1, c_3\}$, Student $s_3$ points to $\{c_2, c_3\}$, Student $s_4$ points to $\{c_3, c_4\}$, Student $s_5$ points to $\{c_4, c_2\}$

Each course meets its capacity, no student is rejected. The algorithm terminates and students are assigned to the courses they are pointing on. The following course allocation is obtained:

\[
\begin{array}{cccccc}
    a_{s_1}^{GS} & a_{s_2}^{GS} & a_{s_3}^{GS} & a_{s_4}^{GS} & a_{s_5}^{GS} \\
    (c_1, c_2) & (c_1, c_3) & (c_2, c_3) & (c_3, c_4) & (c_2, c_4) \\
\end{array}
\]

The following course prices are realised:

\[
P_{c_1}^{GS} = 50, \quad P_{c_2}^{GS} = 30, \quad P_{c_3}^{GS} = 7, \quad P_{c_4}^{GS} = 8
\]
Comparing both outcome, we find the \textbf{Gale-Shapley mechanism} realizes lower course prices in three of four times and improves the course allocation for two students. Both algorithms obtain for student $s_1$, $s_4$ and $s_5$ a similar course allocation. Student $s_2$ and $s_3$ prefer the outcome of the \textbf{Gale-Shapley mechanism}. Especially, student $s_3$ who is assigned via the Point System to only one course gets allocated to her two most preferred courses via the \textbf{Gale-Shapley mechanism}.

The example highlights the \textbf{Gale-Shapley mechanism}’s potential to implement a outcome that pareto-dominates the \textbf{Point System}’s allocation and reduces one opportunity for strategic behaviour by incorporating direct information about students’ preferences.

\textbf{Opportunity for students to cancel matching}

The Sowi Point System enables students to cancel a matching after the publication of course allocations that are implemented by the first and second bidding round. As described in Chapter 3.4, students have to deregister from corresponding courses online before the next round starts. Between first and second enrolment round, students are able to receive a transfer of points including points for dropped courses. Course seats students drop become available in the following round again. There is no possibility for students whose bid was unsuccessful to move up directly in case a seat becomes available which has the potential to lead to a ripple effect of inefficient allocations. The following example illustrates the impact course dropping has.

\textbf{Example 4:}

Suppose the set of students with unfilled slots is $S = \{s_1, s_2\}$. The set of courses with free capacity is $C = \{c_1, c_2, c_3\}$, each course with one free seat remaining. Bid endowment is $E_{s_j, 1} = 11$. As the Point System does not incorporate any direct information about students’ preferences, we will assume truthful bidding. Students allocate their
point endowment upon courses as following:

Student 1 \((s_1)\): \(b_{s_1} = (5, 4, 2)\)

Student 2 \((s_2)\): \(b_{s_2} = (6, 4, 1)\)

The **Sowi Point System’s Algorithm** implements the following course allocation for \(t = 1\):

\[
a^{PS}_{s_1, 1} = c_2 \quad \text{and} \quad a^{PS}_{s_2, 1} = c_1
\]

We assume student \(s_2\) cancels her matching. As student \(s_1\) is enrolled in course \(c_2\), student \(s_2\) only has the opportunity to register in course \(c_3\) in the next enrolment round:

\[
a^{PS}_{s_1, 2} = c_2 \quad \text{and} \quad a^{PS}_{s_2, 2} = c_3
\]

After the second enrolment round, neither student is assigned to her top choice and both student could be made better off by reallocating courses.

As the **Sowi Point System** does not enable students to move up on course seats that become available due to another student’s drop, the opportunity to cancel a matching has the potential to lead to successive inefficiencies (ripple effect). It is conceivably to amplify the course drop period with the opportunity for student to exchange courses in order to enhance the overall outcome.

The **Gale’s Top Trading Cycle** represents an applicable solution concept for reallocation (c.f. Chapter 2.2). After course allocation is implemented for the first bidding round, each student is enabled to point at her most preferred course, i.e. the course she would prefer to be enrolled in comparison to the implemented allocation. The algorithm
of the TTC treats the reallocation as follows:

**Step 1:**
Course $c_2$ points to student $s_1$ whereas student $s_1$ points to course $c_1$. Course $c_1$ points to student $s_2$ whereas student $s_2$ points to $c_2$. There is a cycle without intersections, students are assigned to their choices.

The TTC obtains the course allocation as following:

$$e_{s_1,2}^{TTC} = c_1 \quad \text{and} \quad e_{s_2,2}^{TTC} = c_2$$

Both students prefer the outcome of the reallocation.

The **Gale’s Top Trading Cycle** allowing students to move up on dropped courses cuts the ripple effect. Therefore, the TTC provides an alternative to treat the course-drop period differently and enhances the outcome.

**Härtefallvergabe**

The Sowi Point System incorporates in addition to three bidding rounds another enrolment period named Härtefallvergabe. Therefore, students hand in a course request if they are are not enrolled in four courses at the end of the third enrolment round. Students are allowed to request enrolment in any course. The dean of studies handles requests, he is allowed to add course seats if needed. In general, students are not required to bid consistently over rounds or request in the Härtefallvergabe only courses they have bided on previously.

Students can use this opportunity strategically by bidding on only three courses, i.e. they allocate their endowments on one course less then others, and request their fourth course registration in the Härtefallvergabe. In this way, students manipulate bidding
4. WEAK POINTS

and get an advantaged position. The following example illustrates two different bidding strategies.

Example 5:
Assume there are two students $S = \{s_1, s_2\}$ who both want to get enrolled in the following four courses $C = \{c_1, c_2, c_3, c_4\}$. Both students have the same preference profile: $c_1 \succ c_2 \succ c_3 \succ c_4$. Students are endowed just as in the Point System with $E_{s_1,1} = 1000$, $E_{s_1,2} = 1000$ and $E_{s_1,3} = 1$. We assume truthful bidding. The bidding behaviour over rounds is the following:

<table>
<thead>
<tr>
<th>1st bidding round</th>
<th>2nd bidding round</th>
<th>3rd bidding round</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{s_1,c_1,1} = 400$, $b_{s_1,c_2,1} = 300$</td>
<td>$b_{s_1,c_3,2} = 600$, $b_{s_1,c_4,2} = 400$</td>
<td>$b_{s_1,c_4,3} = 1$</td>
</tr>
<tr>
<td>$b_{s_1,c_3,1} = 200$, $b_{s_1,c_4,1} = 100$</td>
<td></td>
<td>$b_{s_1,c_4,3} = 1$</td>
</tr>
</tbody>
</table>

Stundent $s_2$ requests enrolment in course $c_4$ in the Härtefallvergabe.

Comparing the bidding behaviour, student $s_1$ submits bids for all courses already in the first round and uses additional rounds to bid again for courses if the corresponding bid was unsuccessful. Hence, student $s_1$ uses bids to state her claim and to reflect her preferences, as well. Whereas student $s_2$ uses each round’s endowment to bid only on one single course. The behaviour of student $s_2$ exposes another aspect of the Point System students can misuse for manipulation as the mechanism fails to induce students to reveal preferences fully.

We have already discussed the problem of the bids’ dual role and explained that the **Gale-Shapley mechanism** is promoted to find pareto-dominat outcomes by incorporating bidding behaviour and direct information about students’ preferences. The
4. WEAK POINTS

Gale-Shapley Mechanism presents an alternative creating more efficient outcomes a priori. So the mechanism has the potential to reduce strategic behaviour and post hoc allocation adjustments.

Additional Points for exchange students

Each semester, the University of Innsbruck invites exchange students to join the local studies. Exchange students who participate in a study program listed in Chapter 3.1 enrol in courses via the Sowi Point System. If an exchange student is enrolled in a Socrates program or a Joint Study Program organized with partner universities and registered at the office of International Economics and Business Studies, the student is endowed higher for the first and second enrolment round. The increased endowment for exchange students is supposed to guarantee that exchange students get enrolled in desired courses.

Let $S' = \{s'_1, s'_2, ..., s'_k\}$ with $k \in \{1, 2, ..., k\}$ denote the set of exchange students with a special treatment.

$$S' \subseteq S$$

The students of $S'$ receive the following endowments for the first and second enrolment round:

$$E_{s'_k, t} = 2000 \quad \text{for} \quad t \in \{1, 2\}$$

Consequently, students of $S'$ are enabled to bid in both rounds as following:

$$b_{s'_k, c, t} \in \{1, 2, ..., 2000\} \quad \text{bid of student } s'_k \in S' \quad \text{on } c \in C$$

$$\sum_{c \in C} b_{s'_k, c, t} \leq E_{s'_k, 1} = 2000 \quad \forall \quad b_{s'_k, c, 1} \geq 0$$
Even though exchange students are endowed differently, they participate just as regular students in the Point System. In general, studies’ schedules include mandatory as well as elective courses. Students who bid on courses that are popular among exchange students have an inferior bidding position due to a smaller endowment. As the auction is anonymous regular students are not aware about the conflict and might waste points bidding on such courses. The following example illustrates the conflict.

**Example 6:**

Let us focus on regular student $s_1$ with endowment $E = 1000$ and exchange student $s'_2$ with endowment $E' = 2000$. Both students desire enrolment only in course $c_1$.

The bid of student $s_1$: $b_{s_1,c_1} = 1000$

The bid of student $s'_2$: $b_{s'_2,c_1} = 2000$

Even though the situation is simplified, the discrepancy of unequal endowment and its impact is obvious. Both students allocate their entire endowment on one course. The Point System’s algorithm considers submitted bids in descending order enrolling exchange student $s'_2$ in $c_1$ first. If there is only one seat available, $s_1$ has no chance to be enrolled and due to the lack of information is probably not aware of his disadvantaged position.

In general, bids of regular and exchange students are hardly comparable if students are endowed unequally. By letting all students participate in the course auction, the bidding procedure is lengthened as regular students have to use the additional rounds to get enrolled in other courses. Separate treatment of the exchange students has the potential to create more efficient outcomes via the bidding procedure. In this way, enrolment for exchange students is truly guaranteed and the course catalogue can be updated that the bidding procedure only involves courses regular students are actually able to attain.
The highlighted aspects present an overview of different weak points in the Sowi Point System’s design, serving as examples for students’ strategic behaviour. We show that applying alternative mechanisms has the potential to treat weak points more efficiently and enhance the matching outcome. There are critical points as simultaneous bidding for all courses, partial accumulation of points over rounds and endowment of students whose schedule only needs partially courses included by the Point System, that need to be included into further discussion.
5 CONCLUSION

In this thesis we analysed the Sowi Point System’s bidding mechanism with respect to weak points discussing students’ strategic behaviour and alternative mechanisms. On the basis of the House Allocation Problem as well as the Housing Market we explained desirable properties for mechanism design, i.a. pareto-efficiency and strategy-proofness, and introduce Random Serial Dictatorship and Gale’s Top Trading Cycle as applicable procedures for allocation problems. The literature overview in Chapter 2 also discusses the approach to allocate course seats via course auctions by covering the Standard Bidding Mechanism and the Gale-Shapley Pareto-Dominant Market Mechanism which is promoted by Krishna et al. (2004) as an enhancement of the Standard Bidding Mechanism’s efficiency by inducing a two-sided matching mechanism on the basis of students bidding behaviour and their preference rankings over desired courses. We described the Sowi Point System and its algorithm in Chapter 3, contrasting the Standard Bidding Mechanism by explaining all distinct structures as courses organization with multiple units, the enrolment period’s division into three successive bidding rounds and the Härtefallvergabe. Each enrolment round is mentioned separately with the aim to clarify every step of the process obtaining the final course allocation. In Chapter 4 we combine findings of the theoretical overview with the Point System’s structure exposing the designs weak points. The Point System’s distinct features constitute weak points enabling manipulation of the matching outcome. The current design comprises different weak points which impede a fair and efficient outcome. We emphasize some points by illustrating strategic behaviour of students. Furthermore, we discuss the potential of alternative mechanisms as the Gale-Shapley Mechanism, Random Serial Dictatorship and the Gale’s Top Trading Cycle solving for specific weak points and obtaining improved outcome. Our argumentation supports the research of Krishna et al. (2004) as the Gale-Shapley Mechanism has the potential to improve the Sowi Point System’s bidding outcome by
reducing the loss of efficiency that is a result of the bids dual role. Research and field experiments suggest that the transfer from a bidding mechanism to the Gale-Shapley mechanism can be implemented easily. In this way, the Sowi Point System’s procedure could be shortened and the outcome optimized. It is a debatable point for further discussion how aspects as special treatment for students, the Härtefallvergabe or the opportunity for students’ to cancel a matching should be incorporated into a new design.
References


