Financial Institutions, Markets and Systemic Risk

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1 Motivation and introduction

The financial crisis in 2007/2008 and the subsequent government debt crisis revealed once again that the malfunctioning of the financial system can be disastrous. From an economist’s point of view such events should be studied carefully since they may help the economist to understand and develop solid economic theory. Even more importantly a thoughtful crisis analysis may reveal methods and instruments being able to prevent future crises. This chance motivates my thesis about financial stability and regulation.

In chapter 2 the first essay provides insights regarding the role of banks for the accrual of price bubbles in asset markets and the propagation of liquidity shocks in the financial system. We show that outside investors’ (speculators’) liquidity materially affects the strength of asset price bubbles and contagion in financial markets. Although basically in line with the existing literature, the results of this paper are more general as outside investors’ funding power arises endogenously. In this paper we show that due to an interaction between banks’ business decisions and outside investors’ available amount of funds, at the end of the day, banks dispose of significant steering power regarding the accrual of asset price bubbles and the propagation of shocks in financial markets.

The question about the existence and importance of spill overs across financial institutions is again raised in chapter 3, however from a more econometric perspective. In this paper a novel way of measuring, quantifying and modelling the contagion risk amongst financial institutions is applied. The magnitude of risk spill over effects is gauged by introducing a specific weighting scheme to the regression. This approach originally stems from spatial econometrics. The methodology allows for a decomposition of the credit spread into a contagion risk premium, a systematic risk premium and an idiosyncratic risk premium. The analysis identifies considerable risk spill overs due to the interconnectedness of the financial institutes in the sample. In stress tests, up to one fifth of the CDS spread changes are owing to financial contagion. These results also give an alternative explanation for the nonlinear relationship between a debtor’s theoretical probability of default and the observed credit spreads – known as the “credit spread puzzle”.

Motivation and introduction

The financial crisis has impressively demonstrated the importance of links between entities within the financial system. The malfunction of this – as it turned out highly fragile – network caused severe economic trouble and eventually triggered the fiscal crisis that still holds European economies a grip. The results from the study once again highlight the importance of regulation and supervision – even across legislations. Thus financial market regulations are key for the stability of financial institutions (micro perspective) and the financial system as a whole (macro perspective). Micro and macro prudential regulation must provide an adequate and precise framework in order to guarantee financial stability.

The work discussed in chapter 4 turns the spot light to micro-prudential analysis. In this essay the regulatory regime with respect to interest rate risk for Swiss insurance companies is reviewed in detail and it presents a new approach to measure interest rate risk, which overcomes the shortcomings of the standard model. The standard model of the Swiss Solvency Test is based on more interest rate risk factors than actually needed to capture interest rate risk, it allows for significantly negative interest rates and it tends toward procyclical solvency capital requirements. Our new approach treats interest rate risk with direct reference to the underlying term structure model and interprets its parameters as a canonical choice of the relevant interest rate risk factors. In this way, the number of interest rate risk factors is substantially reduced and interest rate risk measurement is linked to the term structure model itself. The consideration of empirical interest rate data and the acceptance of the economical absurdity of persistently negative interest rates – significantly below the cost of holding cash – motivate the introduction of a truncated Gaussian process to simulate innovation in the future development of the parameters of the underlying term structure model. In a natural way this leads to mean-reverting interest rate behaviour and to countercyclical solvency capital requirements.

The final section chapter 5 concludes and provides an outlook.

Authors: Armin Eder, Falko Fecht, Thilo Pausch

2.1 Introduction

Liquidity shortages of individual banks and subsequent fire-sales led to deteriorating prices during the subprime crisis not only of mortgage-backed securities but also of a broad range of other assets. Similarly, in the sovereign debt crisis tightening refinancing conditions of southern European banks and their offloading of domestic sovereign bonds supposedly contributed to the widening spreads between core countries’ and crisis countries’ sovereign bonds. These asset price drops obviously had severe knock-on effects, for instance, because other financial institutions had to write down their positions held in those assets. However, the extent of these asset price drops strongly depends on the cash-in-the-market, i.e. the ability and willingness of market participants to absorb fire sale of troubled financial institutions (Allen and Gale (1998)). Recent literature has largely emphasized the presence of arbitrageurs (Gromb and Vayanos (2002)) or market makers (Brunnermeier and Pedersen (2009)) in those markets and in particular their financial constraints as a key determinant in markets’ ability to absorb fire sales. Since asset prices, in turn, affect the financial constraints of those market participants, destabilising feedback effects emerge. Those models, however, do not take into account the extent to which the presence of such informed market participants has an effect on the ratio of inside to outside liquidity in the banking system, i.e. on the ratio of assets whose value is common knowledge and state

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1 This essay has been featured in Eder, A., Fecht, F., and Pausch, T. (2014a). Banks, Markets and Financial Stability. *Bundesbank Discussion Paper*, (forthcoming). Furthermore, we were able to present our findings on the Marie Curie Risk Management and Risk Reporting Conference (Berlin, May 2011) and Jahrestagung des Vereins für Socialpolitik (Düsseldorf, September 2013). The very helpful feedback is greatly appreciated.
independent (reserves) and claims against the banking sector that might also serve as a medium of exchange but whose value is endogenous (Bolton et al. (2011)).

In order to endogenize investors’ ability to absorb fire sales, we set up a Diamond/Dybvig-based economy with two banks operating in two separate regions with a continuum of depositors that are only linked through a common asset market. In this asset market, not only banks but also some sophisticated depositors (i.e. arbitrageurs/market makers) can invest. With an exogenous positive probability we assume a run on one bank and study the contagious effect of the subsequent fire sales on the other bank and on equilibrium asset prices. Surprisingly we find that a higher probability of a run not only increases the equilibrium asset price in the normal state (due a liquidity premium) but also leads to a higher asset price in the crisis state. In addition, a higher ratio of sophisticated investors with financial market access may increase financial market depth and improve prices in crisis. It might also increase asset price in no-crisis states.

We obtain these results because asset prices in crisis and no-crisis states are jointly determined by two effects. On the one hand, arbitrage considerations determine asset prices: a lower asset price in crisis periods implies a higher asset price in no-crisis periods. Investors charge a liquidity premium. On the other hand, market prices are settled by cash-in-the-market constraints: the degree to which market participants hold liquidity relative to the fire sales defines the price in crisis states. In our model, liquidity holdings are determined by banks’ reserves which are in turn determined by the optimal deposit contract banks offer. Investors’ financial market access restraints banks’ ability to provide efficient risk sharing against idiosyncratic liquidity shocks as in Diamond (1997) and Fecht (2004). Better financial market access for investors thus reduces banks liquidity holdings. A higher secondary market price for assets in normal times makes investors direct investment opportunities less attractive, improves banks’ liquidity transformation and increases their liquidity holdings. Higher liquidity holdings of banks reduce the asset price drop due to fire sales in crisis periods.

A higher probability of a run with subsequent fire sales and asset price drops increases the liquidity premium market participants charge in normal times. Thus asset prices in normal times increase. A higher asset price in normal states, however, improves banks’ risk sharing capacity – a higher price in normal states makes it less attractive for sophisticated depositors to withdraw their deposits and reinvest in financial markets. It thus fosters banks’ liquidity transformation and increases their liquidity holdings, such that asset prices in crisis times improve.
Relationship to the Literature

A larger share of sophisticated investors with efficient market access improves financial market depth. Consequently cash-in-the-market is higher and price drops following given fire-sales mitigated. This reduces the required liquidity premium in the normal state and thus the asset price in this state. At the same time, however, a higher share of sophisticated investors and thus more liquid financial markets foster banks incentives at a given liquidity premium to rather invest in assets and sell them off in financial markets to gather liquidity rather than to withhold sufficient liquidity ex-ante. In order to ensure that the no-arbitrage condition holds and banks invest in assets and liquidity, the asset price in normal times must increase for more liquid financial markets. Thus if this second effect prevails, more investors having access to financial market might actually lead to higher asset prices in no-crisis periods even though the liquidity premium declines.

In sum, our results explain why banks’ funding liquidity as well as financial market liquidity arises endogenously from the interactions between financial system actors. Both types of liquidity are determined by the ratio of inside to outside liquidity in the banking sector. Banks’ inside liquidity determines available liquidity of both non-sophisticated ans well as sophisticated depositors. Sophisticated investors’ available liquidity, however, crucially drives financial market liquidity and hence asset prices in crisis as well as no-crisis states. Liquidity-driven asset price changes then affect banks’ funding liquidity which ultimately explains why there may be contagion in times of financial crisis. Our results also shed some light on the appearance of asset price bubbles in financial markets. In no-crisis times, the liquidity premium charged by sophisticated investors may be interpreted to determine positive price bubbles while fire-sales prices in times of crisis imply negative price bubbles. Our analysis, in this context, shows relationships between these tow types of bubbles as a consequence of the interactions mentioned previously.

The rest of this chapter is organized as follows. Section 2 relates our paper to the literature. The model is laid out in Section 3. The main analysis is presented in Sections 4 and 5. Section 6 Concludes.

2.2 Relationship to the Literature

Our paper uses [Fecht (2004)] as a baseline for our analysis which in turn builds on the seminal papers of [Diamond and Dybvig (1983), Jacklin (1987), Diamond (1997) and Allen and Gale (2004c)]. Similar to those models, households are exposed to liquidity risk. Banks are able to provide liquidity insurance to households. Financial intermediaries and financial markets coexist. Financial markets allow for trading – and hence liquidating –
claims on long-term investment projects before maturity and may be used by banks as well as households to exchange liquid funds for claims on illiquid (long-term) investment projects. As a result, the model is able to include the aspect of market participation in the analysis of market liquidity.\footnote{Huang and Wang (2008) analyse the effect of market participation on market liquidity and asset price formation in financial markets in more detail.}

Our paper extends the model of Fecht (2004) by considering a positive and commonly known probability of a run on either of the two banks in the financial system. Banks as well as households, therefore, make investment decisions taking into account the possibility of a future bank-run. As a result, optimal decisions may be expected to differ from the standard results of the relevant literature which usually assumes that future crises are not anticipated in the decision-making process, i.e. are zero probability events.\footnote{For example, the papers of Allen and Gale (2000b), Allen and Gale (2004b), Fecht (2004), and Caballero and Simsek (2011) analyse bank behaviour and the propagation of shocks via financial markets. However, all the papers share the common assumption that ex ante the probability of a future financial crisis is zero. As a result, decisions to be made in these papers do not take future financial crises into account.}

Although the assumption is not completely new to the literature\footnote{See, eg, Cooper and Ross (1998) and Freixas et al. (2011).} we are – to the best of our knowledge – the first who consider a positive ex-ante crisis probability in a setting where banks are interconnected via asset markets which also may be directly used by households. Market liquidity, then, arises from the joint effect of household behaviour and bank behaviour. In this way – and in contrast to earlier papers\footnote{See, eg, Cooper and Ross (1998), Holmstrom and Tirole (2000), Bougheas and Ruis-Porras (2005), and Garleanu and Pedersen (2007).} – our setting allows the role of the interaction of funding and market liquidity to be considered as well as conclusions regarding asset price bubbles and financial system stability to be drawn.

These aspects, furthermore, represent the main differences between the present paper and the papers of Allen and Gale (2004a) and Allen and Gale (2000a). Although Allen and Gale (2004a) consider the role of banks regarding the accrual of asset price bubbles, they assume that outside liquidity of speculators is exogenous to the model. Our model overcomes this shortcoming by allowing sophisticated households (which take the role of speculators in our setting) to participate in financial market transactions and to deposit funds with banks. In addition, while in Allen and Gale (2004a) a risky asset return coordinates crisis and no-crisis situations, our model considers an ex ante probability for deposit withdrawals. Asset returns are not exposed to risk. Furthermore, we do not account for agency or information problems as is the case in Allen and Gale (2000a).
The Model

Our paper is, moreover, quite closely related to recent papers of Freixas et al. (2011) and Carletti and Leonello (2011). In particular Freixas et al. (2011) address an objective similar to ours also assuming a non-zero crisis probability. In contrast to our approach, however, they consider direct links between banks via interbank market exposures. In their model, the interbank market redistributes liquidity in the financial system. Moreover, the aggregate amount of liquidity is fixed in Freixas et al. (2011), which is not the case in our model. We consider an asset market (instead of interbank lending) that allows for early liquidation of claims on long-term assets. The market is generally accessible, i.e. households, too, may enter and demand or supply claims on long-term investment projects. As a consequence, in our model, the asset market provides liquidity to market participants who supply claims on long-term assets. And the aggregate amount of liquidity in the market is endogenously determined by the initial decisions of banks and households to invest their funds into short-term or long-term assets.

Just as in our paper, asset market liquidity in Carletti and Leonello (2011) arises endogenously from banks’ initial decisions to invest funds into liquid short-term or illiquid long-term assets. In contrast to our model, the asset market in Carletti and Leonello (2011) is a pure interbank market, i.e. households do not directly have access to the market. Moreover, Carletti and Leonello (2011) do not consider a strictly positive ex-ante probability of a future financial crisis. Instead they focus on the question whether the strength of credit market competition between banks affects bank behaviour and, in turn, financial stability. The aspect of competition between banks is, however, beyond the scope of our paper.

2.3 The Model

Consider a Diamond-Dybvig-style economy with one good, three dates \(t = 0, 1, 2\) and two identical regions \(\{I; II\}\). In each region, there is a continuum of ex-ante identical households of measure 1. A non-random proportion \(\pi\) of households will prefer to consume early – at time \(t = 1\) – and the complementary proportion \(1 - \pi\) will prefer to consume late – at time \(t = 2\). Each household is endowed with one unit of goods and has preferences over consumption \(c_t\) at date \(t = 0\) given by

\[
U(c_1, c_2) = \begin{cases} 
  u_1(c_1) & \text{with probability } \pi \\
  u_2(c_2) & \text{with probability } (1 - \pi)
\end{cases}
\] (2.1)
The uncertainty about the preferred consumption date resolves at \( t = 1 \). This means that every household learns at \( t = 1 \) whether it is patient (prefers consuming at \( t = 2 \)) or impatient (prefers consuming at \( t = 1 \)). However, the individual realization is private information of the respective household and not publicly observable. There is no aggregate uncertainty regarding the share of patient and impatient households. Therefore, from the law of large numbers, the portion of impatient and patient households in the economy as a whole is given by \( \pi \) and \( 1 - \pi \), respectively. For simplicity, there is no discounting and we assume risk-neutral households, i.e. linear utility functions:

\[
U(c_1, c_2) = \begin{cases} 
  x_1 c_1 & \text{with probability } \pi \\
  c_2 & \text{with probability } (1 - \pi) 
\end{cases} \quad (2.2)
\]

\[
x_1 > R \quad (2.3)
\]

In the economy, two different production technologies are available. The first is a pure storage technology that yields zero net interest and enables households to transfer units between any two dates. The second production technology is owned by a continuum of entrepreneurs who do not have any initial endowment but offer a long-run investment project to households. Investments have to be made in \( t = 0 \) to realize some return in \( t = 2 \). At \( t = 1 \), the entrepreneurs decide whether they spend their entire effort and generate a return of \( R > 1 \) at \( t = 2 \) for every unit invested at \( t = 0 \) or whether they shirk. Entrepreneurs have an incentive to reduce their effort since doing so increases their private benefit. Shirking, however, reduces the return of the long-run project to \( \epsilon = 0 \). If the project is prematurely liquidated, it also yields a return of \( \epsilon = 0 \). Table 2.1 summarizes the investment options.

<table>
<thead>
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<th>( t=0 )</th>
<th>( t=1 )</th>
<th>( t=2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Storage</td>
<td>-1</td>
<td>+1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-1</td>
<td>+1</td>
<td></td>
</tr>
<tr>
<td>Production</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>behave</td>
<td>-1</td>
<td>0</td>
<td>R</td>
</tr>
<tr>
<td>shirk</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>liquidate</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.1: Investment options

In order to invest in the long-run project, investors can use a centralized financial market. In \( t = 0 \), households use the primary market to invest in the long-run project by buying

\[6\] Note that \( R \) denotes the return on the long-run technology and \( x_1 \) is the marginal utility of consuming early.
financial claims from an entrepreneur. Since funds are assumed to be scarce, competition between entrepreneurs will lead to a promised repayment of $R$ in $t = 2$. Depending on their consumption needs, households may be inclined to trade the claims on the long-run investment project against consumption goods with other agents in a secondary market in $t = 1$. At $t = 2$, the entrepreneurs pay out the actual return of the project to the final claim holder.

Moreover, households are assumed to be of either two types. A fraction $(1 - i)$ of households is sophisticated (henceforth Type-A). They are able to monitor entrepreneurs and force them to spend their entire effort for the long-term project. Thus these households can realize a return of $R$ on financial claims that they own in $t = 1$. The complementary fraction $i$ of households is of the naive type (henceforth Type-B). They are not able to monitor the entrepreneurs and achieve a return of $\epsilon$ since, then, the entrepreneurs have an incentive to shirk.

Besides the direct investment strategy, consumers can decide to deposit their funds with a bank. A bank is a financial institution that offers deposit contracts against households’ initial endowments. The proceeds from deposit contracts are then used to build up a portfolio containing the investments in the storage technology and claims on the long-term production technology. We assume that only one bank operates in each region. But due to the contestability of the deposit market, both banks are forced to offer households a utility-maximizing deposit contract. Like Type-A households, banks are able to monitor the effort level of entrepreneurs accurately and achieve a return of $R$ on financial claims. But in contrast to sophisticated households, banks are able, through their deposit contract, to credibly commit to pass on the entire return to naive households.\(^7\) Thus, only banks have the ability to provide naive households with efficient access to the long-run investment opportunity.

We further consider two possible states of the world. With strictly positive probability $\theta$, either of the two banks is subject to a run due to a coordination failure of depositors.\(^8\) The probability of such a run is common knowledge to market participants who will adjust

\(^7\) The assumption can be thought of as reflecting Diamond and Rajan (2001) who argue that the attempt to renegotiate the deposit contract would lead to a run on the bank due to sequential service property of deposit contracts (first come, first served).

\(^8\) While we simply assume that such coordination failures occur with an exogenous positive probability, application of global games to Diamond/Dybvig-based models such as Goldstein and Pauzner (2005) show that this can be derived from uncertainty and heterogeneous information about fundamentals.
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their expectations accordingly. Let $m \in M \equiv \{0; 1\}$, where

$$
m = \begin{cases} 
1 & \text{with probability } \theta \\
0 & \text{with probability } (1 - \theta)
\end{cases}
$$

and $\theta \in [0, 1]$ is the probability of a coordination failure state $m = 1$. We assume that $m$ is observable but not verifiable and thus contracts cannot be written contingent on the realization of $m$. Since we have two banks of which one is subject to a run at a time. The probability of a specific bank to be subject to a run is $\theta/2$.

2.4 Financial System Structure and Stability

For an analysis of the structure and stability features of the financial system, assume for the moment that $\theta = 0$. Since household’s type and household’s realized preference shock is the private information, banks are not able to offer contracts contingent on the realization of these characteristics. Thus banks can only offer type-specific deposit contract, if they are self-revealing. A deposit contract specifies depositors’ claims $d_1$ and $d_2$ on the bank at time $t = 1$ and $t = 2$, respectively. If banks offer a deposit contract that provides naive depositors with an option for consumption smoothing, i.e. $d_2/d_1 < R$, then sophisticated households pool with naive households and also choose this contract. Therefore, the optimal deposit contract that banks can offer solves the optimization problem $(P1)$.

\[
\begin{align*}
\max_{l,k} \quad & E[U] = \pi x_1 d_1 + (1 - \pi) id_2 \\
\text{s.t.} \quad & \frac{R}{p_n} d_1 \geq d_2 \quad (IC_A) \\
& d_1 \leq d_2 \quad (IC_B) \\
& d_1 \leq \frac{l + p_n k}{1 - (1 - \pi) l} \quad (BC_1) \\
& d_2 \leq \frac{l - d_1}{1 - \pi} \cdot R \quad (BC_2) \\
& \pi x_1 d_1 + (1 - \pi) \frac{R}{p_n} d_1 > \max \left\{ \pi x_1 + (1 - \pi) \frac{R}{p_n}, \pi x_1 p_n + (1 - \pi) R \right\} \quad (PC)
\end{align*}
\]

As we shall discuss in detail below, sophisticated households always withdraw in equilibrium in $t = 1$. This behaviour is obvious for impatient households but it is also true for patient sophisticated households because, at the equilibrium asset price they withdraw deposits to reinvest directly into asset holdings in $t = 1$. Therefore, sophisticated households choose the deposit contract with the highest $d_1$. This also implies that the bank cannot offer two different type specific contracts that would induce self-revelation as long as the contract meant for naive households provides some consumption smoothing. For detailed proof, see Fecht (2004).
Given that the deposit contract provides some insurance against liquidity risks, sophisticated households, too, might find it optimal to invest in bank deposits in $t = 0$. But in contrast to naive households, sophisticated depositors can withdraw and reinvest in assets in the financial market if they turn out to be patient. While patient naive depositors will have incentives to hold on to their deposits as long as this allows for more consumption at $t = 2$ (see $\text{(IC}_B\text{)}$), patient sophisticated households will rather withdraw their deposits to reinvest in financial markets if this increases consumption in $t = 2$ beyond $d_2$ (see $\text{(IC}_A\text{)}$). Given that they plan to withdraw and reinvest if they turn out to be patient, sophisticated households have an ex-ante incentive to invest in deposits rather than hold a portfolio of liquidity (storage technology) and assets (claims against entrepreneurs) in $t = 0$ and rebalance the portfolio in $t = 1$ according to their consumption preferences and the participation constraint ($\text{(PC)}$).

Given that only naive patient households keep their deposits until $t = 2$, the bank must dispose of sufficient liquidity in $t = 1$ to refinance the repayment $d_1$ to all but the patient naive households. Thus the initial liquidity holding $l$ plus the revenues $p_n \cdot t^{\text{MM}}$ from selling assets in the financial market must suffice to repay $d_1$ to the fraction $[1 - (1 - \pi)i]$ of households. Returns on the long-term asset holdings must suffice to refinance the repayment to patient depositors. Consequently, we have the two budget constraints ($\text{(BC}_1\text{)}$ and ($\text{BC}_2\text{)}$).

Because the banking market is assumed to be contestable, banks will offer a deposit contract that maximizes naive households’ expected utility. Given that $d_2/d_1 < R$ and that sophisticated households withdraw irrespective of whether they are patient or impatient, the deposit contract involves a cross-subsidization, from naive to sophisticated households. Therefore, if a bank does not maximize the expected utility of naive households given this cross-subsidization a competitor could always offer a deposit contract preferable to the naive households leaving the incumbent bank with only the sophisticated households.

Since we assume that banks act as price takers in the financial market, it is easy to see from ($\text{BC}_1$) that for $p_n > 1$ banks will only invest in assets and try to refinance short-term repayments solely with the revenue from asset sales. But this would mean that no liquidity is held in the economy. Thus banks actually could not exchange their assets against liquidity and this cannot be an equilibrium. For $p_n < 1$, banks would only hold liquidity. Patient sophisticated depositors receiving liquidity when withdrawing their

---

10 Note $p_n$ represents the market price for long-term assets when there is no crisis because due to our assumption $\theta = 0$, a possible future crisis is not taken into account by banks and households. Moreover, $k$ represents the amount of long-term assets dedicated to be sold in the market in order to provide the bank with enough liquidity to meet depositor claims. $k$ may, hence, be interpreted as a bank’s trading portfolio or trading book.
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deposits will not find any supply of assets in the market. Thus banks are indifferent only for \( p_n = 1 \) and will sell assets in the financial market while at the same time also investing some of their portfolio in liquidity. Taking this equilibrium asset price into account, it is easy to see that both \((IC_A)\) and \((PC)\) hold for any deposit contract with \( d_1 > 1 \) that provides some liquidity insurance, i.e. \( d_2/d_1 < R \).

Assuming a symmetric equilibrium in which all banks hold the same amount of assets in their trading book, we can derive from the no-arbitrage condition \( p_n = 1 \) the market clearing condition

\[
k = d_1(1 - \pi)(1 - i)
\]

\((MC_n)\)

Given the no-arbitrage condition, we can simplify the budget constraints to

\[
1 \geq (1 - (1 - \pi)i)\ d_1 + (1 - \pi)id_2/R
\]

\((BC)\)

Consequently, as long as the costs of increasing the short-term repayments in terms of forgone long-term repayment are lower than the marginal rate of substitution between short and long-term repayment for naive households, the bank will choose the maximum incentive compatible short-term repayment: whenever the budget constraint is flatter than the indifference curve, the bank will choose \( d_1 \) such that \((IC_B)\) holds with equality, i.e.

\[
x_1 > \frac{1 - (1 - \pi)i}{\pi i} \cdot R
\]

and we have \( d_1 = d_2 \). Reinserting in the budget constraint allows us to derive

\[
d^* = d_1 = d_2 = \frac{R}{R - (1 - \pi)i(R - 1)}
\]

\((2.5)\)

Thus we have the following proposition:

**Proposition 1 (Bank-dominated financial system)** If the fraction of naive households \( i \) is higher than the threshold level \( \hat{i} \) with

\[
\hat{i} = \frac{R}{\pi x_1 + (1 - \pi)R}
\]

banks offer the same short and long-term repayment \( d^* \) on deposits. While this contract allows naive depositors not only to benefit from the long-term productive investment, it also provides them with a maximum liquidity insurance. While sophisticated households,
too, initially deposit their funds with the bank, they withdraw their deposits irrespective of whether or not they are patient or impatient. Patient sophisticated households reinvest the proceeds in the financial market in  \( t = 1 \) buying assets from the banks at the arbitrage-free price \( p_n = 1 \).

If, however, the fraction of naive households is small such that

\[
i < \frac{R}{\pi x_1 + (1 - \pi)R}
\]

the cross-subsidization of sophisticated households becomes too costly for naive depositors. In this case, banks offer a deposit contract

\[
\{d_1; d_2\} = \{1; R\}
\]

which is only attractive for naive households. While this contract allows naive households to benefit from the productive investment, it does not provide any insurance against liquidity risks. Banks (and sophisticated households) are again indifferent between holding assets or liquidity at the arbitrage-free price \( p_n = 1 \). Thus sophisticated households do not fare better investing directly than holding deposits initially. Thus sophisticated households invest directly in liquidity and assets and rebalance their portfolio according to their consumption preference shock. In this case, banks only hold assets to refinance the repayment to patient naive depositors. They do not hold assets to sell them in the financial market.

**Proposition 2 (Market-oriented financial system)** If the fraction of naive households \( i \) is smaller than the threshold level \( \hat{i} \), banks only provide efficient access for naive households to the long-term investment opportunity. Banks do not sell assets in the financial market. Both naive as well as sophisticated households retain considerable liquidity risk.

Now consider in this benchmark case the effects of a run on one bank. In a bank-dominated financial system with \( i > \hat{i} \), the bank affected by the run will not only sell \( k \) assets, this bank is also forced to fire-sale its remaining \( 1 - l - k \) assets. Thus per-capita repayment to depositors is then given by

\[
d_c = l + p_c (1 - l)
\]

(2.6)
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where \( p_c \) represents the fire-sale asset price in the market.

Given that also in a bank-run (i.e. crisis) situation, patient sophisticated depositors will use the repayment to reinvest in the financial market, the market clearing condition would be

\[
(d_c + d_1)(1 - \pi)(1 - i) = p_c [k + (1 - l)]
\]

assuming that the other bank remained solvent and could still repay \( d_1 \) to its patient sophisticated depositors and sell only \( k \) in the financial market. Because \( d_c < d_1 \), the liquidity that patient sophisticated depositors receive from the failing bank and that they use to demand assets in the financial market falls short of the liquidity that they would provide to the asset market if their bank were solvent. At the same time, \( k < (1 - l) \). Consequently, due to the fire sales of the failing banks, asset supply increases while, at the same time, asset demand is being reduced. Thus in the zero probability event of a crisis, the asset price drops to \( p_c < 1 \) in a bank dominated financial system. But from \((MC_n)\) it immediately follows that for \( p_c < 1 \) the other bank does not receive sufficient liquidity from asset sales out of its trading book. Since all of the remaining assets are needed to refinance the repayment to patient naive households, the bank cannot sell additional assets to increase the liquidity inflow. Thus an asset price drop due to one bank’s fire sales cannot be sustained by the other bank and will always lead to contagion in a bank-dominated financial system.

In a market-oriented financial system with \( i \leq \hat{i} \), banks do not rely on liquidity inflow from the financial market. If a run hits one bank and forces it into fire sales, any detrimental effect on asset prices of these fire sales will not destabilize the other bank.

**Proposition 3 (Stability)** If the run on one bank is a zero probability event, this run with subsequent fire sales of assets will lead to an asset price deterioration. In a bank-dominated financial system, the asset price deterioration is unsustainable for the other bank and will inevitably lead to contagion. In a market-oriented financial system, the asset price drop does not affect other banks.

Finally, consider the constrained efficient solution in this setting. The social planner that can shut down financial markets but cannot observe the type of an individual household solves the following problem.

\[
(P^{sp}) \begin{cases} 
\max_{l,k} & E[U] = \pi x_1 c_1 + (1 - \pi) c_2 \\
\text{s.t.} & c_1 \leq c_2 \\
& \pi c_1 + (1 - \pi) \frac{c_2}{R} \leq 1
\end{cases} \tag{IC}
\]

\[
\tag{BC}
\]

14
He maximizes the overall expected utility of naive and sophisticated households. Taking into account only the budget constraint and the incentive constraint ensures that patient households do not withdraw early. For $x_1 > R$, both $(IC)$ and $(BC)$ are binding and the constrained efficient consumption allocation is given by

$$\{c_1^{sp}, c_2^{sp}\} = \left\{ \frac{R}{R - (1 - \pi)(R - 1)}; \frac{R}{R - (1 - \pi)(R - 1)} \right\}$$

Thus the level of risk sharing provided by banks in a bank-dominated financial system ($i > \hat{i}$) is optimal: $d_2/d_1 = c_2/c_1 = 1$. However, the consumption level of patient and impatient naive households is lower than in the optimal allocation because of the information rent extracted by patient sophisticated investors. The difference between the optimal consumption level and the level achieved by naive households in a bank-dominated financial system increases in the share of patient sophisticated households. Sophisticated investors bear considerable liquidity risk. Their consumption level is $d_1$ when patient and $Rd_1$ when impatient. Only if no investors can invest in the financial market ($i = 0$), the allocation achieved by a bank-dominated financial system is optimal.

Sophisticated investors cannot extract an information rent in a market-oriented financial system ($i \leq \hat{i}$). However, in such a system, neither markets nor banks provide the optimal liquidity insurance. The interest rate from $t = 1$ to $t = 2$ is the same in financial markets as in bank deposits. Compared to the constraint efficient allocation banks underinvest in liquidity in both market-oriented and banks-dominated financial systems.

**Proposition 4 (Efficiency)** For $i < 1$, neither the allocation in a market-oriented ($i \leq \hat{i}$) nor in a bank-dominated financial system ($i > \hat{i}$) is constrained efficient. A market-oriented system provides inefficient liquidity insurance. In a bank-dominated financial system, naive households achieve optimal liquidity insurance but pay an information rent to patient sophisticated investors. The larger this information rent, i.e. the larger the share of patient sophisticated investors $(1 - \pi)(1 - i)$, the less efficient the allocation in a bank-dominated financial system.

### 2.5 Asset Price Bubbles

Consider now the run on one bank as an event that occurs with a small but positive probability. In a bank-dominated financial system, a run on one bank and the resulting fire sales will always induce a liquidity shortage at the other bank unless banks hold
liquidity buffers. However, as long as the run on one bank occurs with a sufficiently low probability, the expected costs of holding liquidity buffers to avoid contagion, i.e. the reduced repayment on deposits in a no-crisis state, overcompensate the expected benefits from being able to sustain the asset price drop following fire sales of the other bank. Thus for a sufficiently low \( \theta \), the possibility of a liquidity crisis will only affect asset prices. The resulting price volatility, however, turns out to be more extensive than just an asset price drop in times of crisis. In normal times, the asset price can be shown to include a liquidity risk premium in order to compensate banks for the liquidity risk they incur, i.e. for the expected costs of contagion through financial markets. As a result, the present situation shows that asset price bubbles accrue when banks and households consider a small but strictly positive crisis probability.

Let \( \bar{\theta} \) denote some threshold crisis probability up to which a run on one bank is sufficiently unlikely (\( \theta \leq \bar{\theta} \)). Then, banks can still provide some consumption smoothing for naive households \( R > d_2 > d_1 > 1 \) (i.e. the fraction of naive households is again sufficiently high \( i \geq \bar{i} \)) banks offer a deposit contract that solves the optimization problem \( (P2) \).

\[
\begin{align*}
\max_{l,k} \quad & E[U] = (1 - \theta) \left[ \pi i x_1 d_1 + (1 - \pi) i d_2 \right] + \theta \left[ \pi i x_1 d_c + (1 - \pi) i d_c \right] \\
\text{s.t.} \quad & \frac{R}{p_n} d_1 \geq d_2 \quad (IC_A) \\
& d_1 \leq d_2 \quad (IC_B) \\
& d_1 \leq \frac{l + p_c k}{1 - (1 - \pi)^i} \quad (BC_1) \\
& d_2 \leq \frac{(1 - l - k)}{(1 - \pi)^i} \cdot R \quad (BC_2) \\
& d_c \leq (1 - l) p_c + l \quad (BC_c) \\
& (1 - \theta) \left[ (1 - \pi) \frac{R}{p_n} d_1 + \pi x_1 d_1 \right] + \theta \left[ (1 - \pi) \frac{R}{p_c} d_c + \pi x_1 d_c \right] \quad (PC) \\
& > \max \left\{ (1 - \pi) \left[ \theta \frac{R}{p_c} + (1 - \theta) \frac{R}{p_n} \right] + \pi x_1, \\
& \pi x_1 \left[ \theta p_c + (1 - \theta) p_n \right] + (1 - \pi) R \right\}
\end{align*}
\]

When designing the optimal deposit contract, banks must also take into account the amount that they can repay in a crisis if such an event has a positive probability. Due to the financial market activity, in a bank-dominated financial system not only the bank that directly suffers from a run is forced to liquidate its entire portfolio in the market, the other bank will be liquidated because of a liquidity shortage given that it does not hold a liquidity buffer. Consequently, in the depositors’ expected utility function that banks maximize, we only need to consider the banks’ repayment \( d_1 \) and \( d_2 \) to patient and impatient depositors when there is no crisis and the per-capita liquidation return \( d_c \) that both banks can distribute in the crisis.
While the incentive compatibility constraints for naive and sophisticated households \((IC_A)\) and \((IC_B)\) and the budget constraints for early and late repayment \((BC_1)\) and \((BC_2)\) remain unchanged, we also need to take into account the budget constraint \((BC_c)\) for the crisis situation. This constraint simply states that the repayment per capita after liquidation equals at most the entire liquidation proceeds whereby all assets are sold off in the financial market at the crisis price \(p_c\).

Finally, in contrast to the no-crisis case, the participation constraint of sophisticated depositors \((PC)\) must now take into account that prices in the asset market and the repayment of banks both vary depending on the different states that can occur. Thus it is only preferable for sophisticated depositors to initially invest in deposits if the expected payoff, that they can realize by withdrawing and consuming if impatient or reinvesting in financial markets if patient is larger than the payoff they realize by investing either only in liquidity or assets and trade in the financial market in \(t = 1\) according to their realized consumption preferences.

In addition it is worth mentioning that from the budget and participation constraints above, one can observe in which way a bank’s inside liquidity affects the amount of sophisticated investors’ outside liquidity. In a no-crisis situation, a high market price \(p_n\) supports banks’ inside liquidity which, in turn, maintains high repayments \(d_1 > d_c\). As a result, in a no-crisis situation, sophisticated households dispose of plenty of liquidity which can be reinvested in the asset market keeping asset prices high. In contrast, in times of crisis, the low market price \(p_c\) reduces sophisticated investors’ funds due to low repayment on deposit contracts \(d_c < d_1\). This, in turn, limits market liquidity and puts further strain on market prices.

Therefore, supplementary to the optimal deposit contract solving \((P2)\), the equilibrium with a bank-dominated financial system and an infrequent crisis is characterized by the market clearing condition for the asset market in the good and in the bad state. In the no-crisis state, the market value of the bank’s trading portfolio must be equal to the withdrawals of patient sophisticated households who reinvest in financial markets.

\[
p_n k = d_1 (1 - \pi) (1 - i) \quad (MC_n)
\]

In the crisis situation, the market value of the entire asset holding, i.e. the trading book plus the banking book, must equal the cash received by the patient sophisticated households from the liquidation of their respective bank.
Because of the higher marginal utility of impatient depositors, depositors’ expected utility in the no-crisis state is optimized with a maximum repayment on deposits in the short-run for $p_n \geq 1$. Taking the incentive constraints of patient naive households into account, maximum expected utility for the no-crisis period is achieved with $d_1 = d_2$. In the crisis state, depositors’ utility is maximized with a maximum $l$ for $p_c \leq 1$. Increasing the liquidity holdings beyond the amount required to implement $d_1 = d_2$ is costly in the no-crisis state because holding such a liquidity buffer would imply that the repayment to patient depositors in the no-crisis state is inefficiently refinanced with proceeds from the storage technology rather than the long-term investment technology. Consequently, it is efficient for the bank not to increase its liquidity holdings beyond what is needed to implement the optimal repayments in the no-crisis state if the marginal disutility from holding a liquidity buffer in the no-crisis state is not smaller than the benefits in the crisis period

$$
(1 - \theta) i [\pi x_1 + (1 - \pi)] \frac{(R - 1)}{(1 - \pi) i + (1 - (1 - \pi) i) R} \geq \theta i [\pi x_1 + (1 - \pi) i] (1 - p_c)
$$

Thus, as long as the crisis probability is lower than a threshold $\tilde{\theta}$ with $^{11}$

$$
\tilde{\theta} = \frac{(R - 1)}{[R - (1 - \pi) i (R - 1)] (1 - p_c) + (R - 1)}
$$

banks will not hold excess liquidity and will choose a portfolio to maximize depositors’ expected utility in the no-crisis state.

Taking as given that prices $p_n$ and $p_c$ adjust such that banks are indifferent between holding liquidity or investing in assets banks hold in equilibrium exactly enough assets in their trading book such that $(MC_n)$ holds. The withdrawals of all impatient depositors must be financed with liquidity holdings and the repayment to patient naive households who only withdraw their depositors in $t = 2$ will be financed out of the banking book, i.e. assets held until maturity. Since $(IC_b)$ is the binding constant, it will hold with equality. Thus given $\theta \leq \tilde{\theta}$, the optimal repayment in no-crisis situations is given by the general budget constraint

Note that this implies that $p_c \geq 1 - \frac{(1-\theta)}{\theta} \frac{(R-1)}{[R - (1 - \pi) i (R - 1)]}$. If $p_c$ drops below this threshold, banks would find it beneficial to only invest in liquidity. However, if banks only invest in liquidity, a bank-dominated financial system does not emerge and banks are redundant.
\[(1 - i)(1 - \pi) d^*/p_n + \pi d^* + i (1 - \pi) d^*/R = 1\]

Consequently, the optimal deposit contract is given by

\[d^{**} = d_1 = d_2 = \frac{p_n R}{(1 - \pi) [(1 - i) R + ip_n] + \pi p_n R} \tag{2.7}\]

and banks’ liquidity holding is

\[l^{**} = \frac{\pi p_n R}{(1 - \pi) [(1 - i) R + ip_n] + \pi p_n R} \tag{2.8}\]

Obviously, both banks’ liquidity holdings as well as their repayments in no-crisis times increase in the asset price in no-crisis states.

Inserting (2.8) and (BC\(_c\)) from (P2) in the market clearing condition for the crisis period (MC\(_c\)) gives the following cash-in-the-market equilibrium condition for the asset price in the crisis period\(^\text{12}\)

\[p_c = \frac{\pi p_n R}{[(1 - i) R + ip_n] \cdot \frac{(1 - i)}{(1 - (1 - \pi)(1 - i))}} \tag{CMP}\]

which implies that the asset price in the crisis state increases in the price in normal times

\[\frac{\partial p_c}{\partial p_n} = \frac{\pi}{(1 - (1 - \pi)(1 - i))} \cdot \left(\frac{R (1 - i)}{[(1 - i) R + ip_n]}\right)^2 > 0 \tag{2.9}\]

The intuition for this is that the larger the price in the no-crisis state, the larger the general repayment that banks can afford in no-crisis times. To fund the higher repayment for impatient households, banks hold somewhat more liquidity. In the crisis state, a larger liquidity holding reduces the amount of assets being sold in the market and reduces asset price deterioration during the banking crisis\(^\text{13}\)

\[d^{**}_c = \frac{\pi p_n R}{(1 - \pi)[(1 - i) R + ip_n] + \pi p_n R} \cdot \frac{1}{1 - (1 - \pi)(1 - i)} \tag{2.10}\]

The condition that we have not considered so far but that is required to close the model is the no-arbitrage condition. An equilibrium combination of asset prices in normal and

\(^{12}\) See Appendix A for details.

\(^{13}\) See Appendix A for details.
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crisis times requires that banks are ex ante indifferent between investing in assets or holding liquidity.

If the bank only holds liquidity, it could repay all early withdrawing depositors, both impatient and patient sophisticated ones, with liquidity and use liquidity to buy assets at the no-crisis price $p_n$ to finance the repayments to impatient naive depositors. Following that strategy, the bank could pay depositors in $t = 1$ and $t = 2$

$$d = \frac{R}{(1 - (1 - \pi)i)R + (1 - \pi)ip_n}$$

Since a bank holding only liquidity could repay $d_c = 1$ in the crisis, the expected utility a bank following that strategy could provide to naive households is given by

$$(1 - \theta) [\pi x_1 + (1 - \pi)]i \frac{R}{(1 - (1 - \pi)i)R + (1 - \pi)ip_n} + \theta [\pi x_1 + (1 - \pi)]i \quad (2.11)$$

A bank that only invests in assets and sells some of them off in $t = 1$ to finance the short-term repayments would be able to pay

$$d = \frac{R}{(1 - (1 - \pi)i)\frac{R}{p_n} + (1 - \pi)i}$$

Given that during a crisis the bank would have to sell off all its assets at the equilibrium price $p_c$, expected utility of naive households depositing at a bank that only invests in asset amounts to

$$(1 - \theta) [\pi x_1 + (1 - \pi)]i \frac{R}{(1 - (1 - \pi)i)\frac{R}{p_n} + (1 - \pi)i} + \theta [\pi x_1 + (1 - \pi)]ip_c \quad (2.12)$$

Thus from the equality of (2.11) and (2.12) follows that banks will be indifferent between holding liquidity and investing in assets given the following no-arbitrage condition

$$p_c = 1 - \frac{(p_n - 1)R}{\theta \left(\frac{p_n - 1)R}{(1 - (1 - \pi)i)R + (1 - \pi)ip_n}\right)} \quad (NAC)$$

Following the no-arbitrage condition, the equilibrium asset price in crisis states is a decreasing function of the asset price in no-crisis states

---

14 See Appendix A for details.
\[
\frac{\partial p_c}{\partial p_n} = \frac{(1 - \theta)}{\theta} \frac{(1 - (1 - \pi)i)R^2 + (1 - \pi)iR}{[(1 - (1 - \pi)i)R + (1 - \pi)i\pi n]^2} < 0
\] (2.13)

Intuitively, a higher asset price in normal times makes asset holdings more attractive. In order to ensure that banks are indifferent, the price in crisis periods must be lower.

Using [CMP] and [NAC] finally allows us to determine the equilibrium asset price in no-crisis and crisis states. From (2.9) it is easy to see, that according to [CMP], \( p_c \) is a monotonically increasing concave function in \( p_n \) \( \forall p_n \in \mathbb{R}_{\geq 0} \), while (2.13) indicates, that \( p_c \) according to [NAC] is a monotonically decreasing convex function of \( p_n \) \( \forall p_n \in \mathbb{R}_{\geq 0} \). Consequently, there is only one equilibrium combination of asset prices. Figure 2.1 illustrates the case.

Taking a closer look at CMP and NAC, furthermore, shows that besides fundamentals \( R \) and \( \pi \) of the model, the level of the ex-ante crisis probability \( \theta \) together with the share of naive households \( i \) in particular determine the unique equilibrium combination of asset prices \( (p_n, p_c) \).

Consider first the role of \( \theta \) and note that CMP is independent of \( \theta \). Thus, as depicted in Figure 2.1] CMP expressing \( p_c \) as a function of \( p_n \) does not change if \( \theta \) varies. NAC, to the contrary, is affected by a change in \( \theta \). An increase in \( \theta \) increases the coefficient of \( p_n \) in NAC. Thus, in Figure 2.1] NAC expressing \( p_c \) as a function of \( p_n \) is turned to the upper right moving asset prices \( (p_{n,1}, p_{c,1}) \) in crisis as well as normal states to a higher level \( (p_{n,2}, p_{c,2}) \). Thus, as depicted in Figure 2.1] and panel a) of Figure 2.3] an increase in the crisis probability \( (\theta) \) leads to soaring prices in both the crisis as well as the no-crisis state.

![Figure 2.1: Impact of a change in \( \theta \) on equilibrium prices](image)

Intuitively, for a given asset price in crisis times, an increase in the crisis probability increases the required liquidity premium. Thus the asset price in normal times must rise
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(shift of NAC). However, a shift of the asset price in normal times improves banks’ ability to meet the asset demand of patient sophisticated depositors. This, in turn, increases overall repayments on deposits and banks’ liquidity holdings. As a consequence, the asset price in crisis periods drops less sharply than in the case of less frequent crises.

**Proposition 5 (Asset Price Bubbles)** For \( \theta \leq \min\{\bar{\theta}, \tilde{\theta}\} \) there is only one equilibrium related to one combination of asset prices \((p_n, p_c)\). The larger \( \theta \) in the considered interval, the larger both \( p_n \) and \( p_c \).

**Proof 1** See Appendix B.

In sum, in a bank-dominated financial system, a higher probability of a run on one bank with associated fire sales and contagion of the other bank leads to overall higher asset prices in normal times due to a higher required liquidity premium. A higher level of \( \theta \) and, hence, a higher asset price increases the overall value of liquidity which, in turn, induces banks to hold more liquidity. Thus more can be paid for assets both in the crisis as well as in the no-crisis state.

From (2.7) and (2.10) it is straightforward to see that a higher asset price in the no-crisis state increases the repayment investors receive. The intuition is that with a higher asset price, the information rent that patient sophisticated investors can extract is lower. Therefore, the repayment that banks can provide to depositors is higher. Banks have invested fewer resources ex ante in assets that are only held to sell that to patient sophisticated investors in the market. Thus the liquidity insurance provided by the banking sector in the no-crisis state becomes more effective and approaches the constant efficient allocation. Thus the threat of a crisis with the fostered incentives to withhold liquidity improves the efficiency of the deposit contract and the allocation achieved if banks are stable.

The proposition also sheds some light on the role of banks’ liquidity transformation in the accrual and strength of asset price bubbles. Banks withhold reserves, i.e. *outside liquidity*, to repay impatient depositors. But they also create *inside liquidity*. They issue claims, deposits, that can be used as a medium of exchange. In particular, patient sophisticated depositors use their deposits to pay for the assets they buy from banks in the normal state. However, in a banking crisis inside liquidity loses its value and claims against banks are only worth the reserves held by banks to back their deposits. Thus while in normal times sophisticated investors can use the full value of their deposits to buy assets, in crisis states their ability to absorb fire sales of assets is determined by the outside liquidity
Asset Price Bubbles

of the banking sector. When there is no crisis, there is ample liquidity in the system maintaining asset prices at very high levels. In a crisis situation, declining asset prices reduce the inside liquidity in the banking system and scarcity of liquid funds deteriorates asset prices even further. This downward spiral leads to financial contagion and financial instability.

The impact of \( i \) on the equilibrium combination of asset prices, however, is less straightforward. Because both CMP and NAC depend on \( i \), a change in the share of naive households affects both functions. Nevertheless, it is still straightforward to see that \( p_c \) decreases with an increase of naive households. First, observe that NAC as well as CMP decrease with \( i \).

\[
\frac{\partial NAC}{\partial i} = -\frac{(1-\theta)}{\theta} \left( \frac{(1-\pi)(R-p_n)(p_n-1)R}{((1-(1-\pi)i)R+(1-\pi)ip_n)^2} \right) < 0
\]

\[
\frac{\partial CMP}{\partial i} = \left( \frac{\pi p_n R}{(1-i)} \right) \left( \frac{(1-i)R + ip_n}{((1-\pi)R+(1-\pi)(1-i))^2} \right) \frac{1}{\pi p_n R} < 0
\]

Since both function decrease with \( i \), the equilibrium asset price \( p_c \) has to decrease as well. The impact of \( i \) on \( p_n \) is ambiguous which prevents a clear conclusion about the relationship between \( i \) and the equilibrium combination of asset prices \((p_n, p_c)\). In Figure 2.2 we plot the three possible consequences of change in the fraction of naive households on the equilibrium asset prices. In each plot we increased the fraction of naive households from \( i_1 \) to \( i_2 \). The single difference between these plots is the impact of \( \Delta i \) on NAC and CMP, where we define \( \Delta NAC(p_n) = abs(NAC(p_{n,1}, i_2) - NAC(p_{n,1}, i_1)) \) and \( \Delta CMP(p_n) = abs(CMP(p_{n,1}, i_2) - CMP(p_{n,1}, i_1)) \). In panel (a) of Figure 2.2, the impact of an increase in \( i \) on NAC is larger in comparison to the impact on CMP, \( \Delta NAC(p_n) > \Delta CMP(p_n) \), while in panel (c) the opposite is true, \( \Delta NAC(p_n) < \Delta CMP(p_n) \). Finally, panel (b) illustrates the case \( \Delta NAC(p_n) = \Delta CMP(p_n) \). In each case, \( p_{c,2} \) is smaller than \( p_{c,1} \) supporting the conclusion that \( p_c \) is a decreasing function with respect to \( i \). In sharp contrast, the effect on \( p_n \) critically depends on difference \( h(p_n) := \Delta NAC(p_n) - \Delta CMP(p_n) \). If \( h(p_n) > 0 \), \( p_{n,1} > p_{n,2} \); hence \( p_n \) is a decreasing function of the fraction of naive households. This situation is depicted in panel (a) of Figure 2.2. If the opposite is true and \( h(p_n) < 0 \), \( p_n \) increases with \( i \), which is shown in panel (c). Finally, if \( h(p_n) = 0 \), \( p_n \) is not affected by an increase in the fraction of naive households (panel b). This brings the

15 The function \( abs(x) \) is defined by \( abs(x) = \begin{cases} x & \text{für } x \geq 0 \\ -x & \text{für } x < 0 \end{cases} \).
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Figure 2.2: Comparative statics of equilibrium asset prices with respect to $i$

following conclusion

$$\frac{\partial p_n}{\partial i} = \begin{cases} > 0 & \text{if } \frac{\partial \text{NAC}(p_n)}{\partial i} > \frac{\partial \text{CMP}(p_n)}{\partial i} \\ < 0 & \text{if } \frac{\partial \text{NAC}(p_n)}{\partial i} < \frac{\partial \text{CMP}(p_n)}{\partial i} \\ = 0 & \text{if } \frac{\partial \text{NAC}(p_n)}{\partial i} = \frac{\partial \text{CMP}(p_n)}{\partial i} \end{cases}$$

The sign of $\frac{\partial p_n}{\partial i}$ is determined by the fraction of impatient households that determine.

$[\frac{\partial \text{NAC}}{\partial i}]_{\pi=0} < 0$ while $[\frac{\partial \text{CMP}}{\partial i}]_{\pi=0} = 0$, hence $[\frac{\partial \text{NAC}}{\partial i} - \frac{\partial \text{CMP}}{\partial i}]_{\pi=0} < 0$. On the contrary, $[\frac{\partial \text{NAC}}{\partial i}]_{\pi=1} = 0$ while $[\frac{\partial \text{CMP}}{\partial i}]_{\pi=1} < 0$, hence $[\frac{\partial \text{NAC}}{\partial i} - \frac{\partial \text{CMP}}{\partial i}]_{\pi=1} > 0$. Consequently, the total effect of $i$ on $p_n$ depends critically on the fraction of early consuming households $\pi$.

For a formal proof, see Appendix B.
This is shown graphically in panel (b) and (c) of Figure 2.3. Panel (b) shows a financial system with a relatively small fraction of impatient households, $\pi = 0.2$. The same financial system is depicted in panel (c), yet with a high fraction of impatient households $\pi = 0.7$. It turns out that in the first financial system $p_n$ is not a monotonic increasing function with respect to $i$, while in the second financial system $p_n$ is a strictly increasing function.

![Graph showing comparative statics of equilibrium asset prices](image)

**Figure 2.3: Comparative statics of equilibrium asset prices**

**Proposition 6 (Asset Price Bubbles)** For $\theta \leq \min\{\bar{\theta}, \tilde{\theta}\}$ the unique equilibrium asset price in crisis $p_c$ declines in $i$, while a change in $i$ has an ambiguous effect on $p_n$. For a sufficiently high ratio of impatient households the asset price in normal times $p_n$ also declines.

**Proof 2** See Appendix B. ■

The intuition for this result is somewhat subtle. For a given $p_n$, an increase in the fraction of naive households decreases financial market depth, exacerbates the cash-in-the-market constraint and contributes to a lower asset price during a crisis (CMT turns to the lower right). This effect is the stronger the larger the ratio of patient households. At the same time, for a given asset price in normal times $R > p_n > 1$ it is less preferable for banks to invest in assets and sell them off in financial markets to acquire the liquidity needed to repay patient sophisticated and impatient depositors the smaller their share of those households in the population. Since the smaller their share, the relative arbitrage profit they can make with this strategy compared to investing in liquidity in $t = 0$ is smaller. Therefore, the smaller the share of (patient) naive households, the more banks could
increase the repayment on deposits in normal times. Thus for a given $p_n$, the incentives to invest only in assets decrease in $i$. To compensate for this, the price drop in crisis periods must be smaller (NAC turns to the upper right).

### 2.6 Conclusion

In this paper we endogenize the liquidity risk sharing provided by the banking sector and the ratio of outside (reserves) to inside liquidity (claims against the banking sector) held in the financial system. Outside liquidity held by the financial system is a key determinant of sophisticated investors’ (arbitrageurs or speculators) ability to absorb fire sales of run-prone financial institutions. At the same time, sophisticated investors ability to buy assets in financial market affects banks’ risk sharing and their holdings of inside relative to outside liquidity. Thus asset prices in crisis and no-crisis states are jointly determined by a cash-in-the-market pricing and a no-arbitrage condition.

This set-up allows us to show that a higher probability of a run on one bank increases the asset prices in the crisis as well as in the normal state. This result is driven by the interaction of the cash-in-the-market pricing and the no-arbitrage condition: a higher crisis probability increases the probability that a bank has to sell off its assets at a fire-sale price. Thus to be compensated for these losses, the no-arbitrage condition requires a higher no-crisis price for the assets – a liquidity premium. But a higher asset price in normal times improves banks’ ability to provide liquidity insurance. Thus they hold more outside liquidity, which fosters the ability of sophisticated investors (arbitrageurs) to absorb fire sales. The cash-in-the-market pricing is alleviated.

Better access to financial markets increases market depth. Consequently, the price in the crisis state increases with the fraction of sophisticated households. This reduces the liquidity premium charged in the normal state. However, more liquid financial markets also foster banks’ incentives to sell off asset holdings them off in financial markets in order to gather liquidity instead of holding sufficient liquidity ex ante. Thus the no-arbitrage condition requires that prices in the normal state increase. Thus an improved financial market access has an ambiguous effect on the price in the normal state.

Our results shed some light on the potential value of regulatory liquidity requirements – although not explicitly considered in the model. In our model liquidity regulation, which defines some minimum liquidity buffer, may be expected to strengthen financial stability at the cost of a less efficient allocation of funds in normal times. A sufficiently large mandatory liquidity buffer might enable banks that are not affected by a run to
sustain asset price drops resulting from fire sales of other banks. Moreover, liquidity requirements will alleviate the cash-in-the-market constraint. At the same time, though, a higher liquidity requirement directly impairs the efficiency of banks’ liquidity insurance. Moreover, a lower liquidity premium will lead to a lower asset price in normal times which further deteriorates the efficiency of banks’ liquidity transformation.

Moreover, central bank interventions, such as the Bernanke Put in the financial crisis of 2007-2009, help to stabilize the financial system by mitigating asset price drops. Conditional liquidity injections will relax cash-in-the-market constraints. Hence asset prices in the crisis state will increase and prices in normal times will decline due to a lower liquidity premium. This, however, again impairs banks’ ability to provide an efficient risk sharing and thus bears some efficiency losses.
Appendix A

Equilibrium conditions given infrequent crisis

Deriving the crisis price: Inserting (2.8) and \((BC_c)\) from \((P2)\) in the market clearing condition for the crisis period \((MC_c)\) yields

\[
p_c(1 - l) = [(1 - l)p_c + l] (1 - \pi)(1 - i)
\]

\[
p_c(1 - l) (1 - (1 - \pi)(1 - i)) = l(1 - \pi)(1 - i)
\]

\[
p_c = \frac{l}{(1 - l)} \cdot \frac{(1 - \pi)(1 - i)}{(1 - (1 - \pi)(1 - i))} \quad (2.14)
\]

\[
\frac{l}{(1 - l)} = \frac{\pi p_n R}{(1 - \pi)[(1 - i) R + ip_n] + \pi p_n R} = \frac{\pi p_n R}{(1 - \pi)[(1 - i) R + ip_n]} \quad (2.15)
\]

Inserting (2.15) in (2.14) gives

\[
p_c = \frac{\pi p_n R}{(1 - \pi)[(1 - i) R + ip_n]} \cdot \frac{(1 - \pi)(1 - i)}{(1 - (1 - \pi)(1 - i))}
\]

\[
p_c = \frac{\pi p_n R}{(1 - \pi)[(1 - i) R + ip_n]} \cdot \frac{(1 - i)}{(1 - (1 - \pi)(1 - i))}
\]

Deriving the per-capita repayment in crisis: Inserting \((\text{CMP})\) and (2.8) in \((BC_c)\) from \((P2)\) yields

\[
d_c = \left(1 - \frac{\pi p_n R}{(1 - \pi)[(1 - i) R + ip_n] + \pi p_n R} \right) \frac{\pi p_n R}{(1 - \pi)[(1 - i) R + ip_n] + \pi p_n R}
\]

\[
d_c = \frac{(1 - \pi)[(1 - i) R + ip_n]}{(1 - \pi)[(1 - i) R + ip_n] + \pi p_n R} \cdot \frac{\pi p_n R}{(1 - \pi)[(1 - i) R + ip_n]} \cdot \frac{(1 - i)}{(1 - (1 - \pi)(1 - i))} +
\]

\[
+ \frac{(1 - \pi)[(1 - i) R + ip_n]}{(1 - \pi)[(1 - i) R + ip_n] + \pi p_n R} \cdot \frac{\pi p_n R}{(1 - \pi)[(1 - i) R + ip_n]} \cdot \frac{(1 - i)}{(1 - (1 - \pi)(1 - i))} +
\]

\[
+ \frac{\pi p_n R}{(1 - \pi)[(1 - i) R + ip_n] + \pi p_n R} \cdot \frac{(1 - \pi)(1 - i)}{(1 - (1 - \pi)(1 - i))} + \frac{\pi p_n R}{(1 - \pi)[(1 - i) R + ip_n] + \pi p_n R}
\]
\[ d_c = \frac{\pi p_n R}{(1 - \pi) \left[ (1 - i) R + ip_n \right] + \pi p_n R} \cdot \left( 1 + \frac{(1 - \pi)(1 - i)}{(1 - (1 - \pi)(1 - i))} \right) \]

\[ d_c = \frac{\pi p_n R}{(1 - \pi) \left[ (1 - i) R + ip_n \right] + \pi p_n R} \cdot \frac{1}{1 - (1 - \pi)(1 - i)} \]

**Deriving the no-arbitrage condition:** If the bank only holds liquidity it could repay all early withdrawing depositors impatient and patient sophisticated ones with liquidity and use liquidity to buy assets at the no-crisis price \( p_n \) to refinance the repayments to impatient naive depositors. As a consequence, when only holding liquidity, a bank would face the budget constraint

\[
\pi d + (1 - \pi) (1 - i) d + (1 - \pi) \frac{p_n}{R} d = 1
\]

\[
\left[ \pi + (1 - \pi) (1 - i) + (1 - \pi) \frac{p_n}{R} \right] d = 1
\]

Following that strategy, the bank could pay depositors in \( t = 1 \) and \( t = 2 \)

\[
d = \frac{1}{\pi + (1 - \pi) (1 - i) + (1 - \pi) \frac{p_n}{R}}
\]

Since holding only liquidity permits the bank to pay \( d_c = 1 \) in the crisis period, the expected utility that a bank could provide to naive households would be

\[
(1 - \theta) \left[ \pi x_1 + (1 - \pi) \right] \frac{1}{\pi + (1 - \pi) (1 - i) + (1 - \pi) \frac{p_n}{R}} + \theta \left[ \pi x_1 + (1 - \pi) \right] i
\]

A bank that only invests in assets and sells some of them off in \( t = 1 \) to refinance the short-term repayments would be able to repay

\[
d = \frac{1}{\pi \frac{p_n}{p_n} + \frac{(1 - \pi)(1 - i)}{p_n} + (1 - \pi) \frac{1}{R}}
\]

Given that during a crisis the bank would have to sell off all its assets at the equilibrium price \( p_c \), the expected utility of naive households depositing at a bank that only invests in asset amounts to

\[
(1 - \theta) \left[ \pi x_1 + (1 - \pi) \right] \frac{1}{\pi \frac{p_n}{p_n} + \frac{(1 - \pi)(1 - i)}{p_n} + (1 - \pi) \frac{1}{R}} + \theta \left[ \pi x_1 + (1 - \pi) \right] ip_c
\]
Thus banks will be indifferent between holding liquidity and investing in assets if

\[
(1 - \theta) \frac{\pi}{p_n} + \frac{(1-\pi)(1-i)}{p_n} + (1 - \pi) i \frac{1}{R} + \theta p_e = (1 - \theta) \frac{1}{\pi + (1 - \pi)(1 - i) + (1 - \pi) i \frac{p_n}{R}} + \theta
\]

\[
p_e = \frac{(1 - \theta)}{\theta} \left( \frac{1}{\pi + (1 - \pi)(1 - i) + (1 - \pi) i \frac{p_n}{R}} - \frac{1}{\pi + (1 - \pi)(1 - i) + (1 - \pi) i \frac{p_n}{R}} + 1 \right)
\]

\[
p_e = 1 - \frac{(1 - \theta)}{\theta} \left( \frac{p_n - 1}{\pi + (1 - \pi)(1 - i) + (1 - \pi) i \frac{p_n}{R}} \right)
\]
Appendix B

Partial differentials of CMP and NAC

**CMP**

\[
CMP := p_c = \frac{\pi p_n R}{(1 - i) R + ip_n} \cdot \frac{(1 - i)}{(1 - (1 - \pi)(1 - i))} \quad \text{(CMP)}
\]

This gives us the following partial derivatives

\[
\begin{align*}
\frac{\partial CMP}{\partial p_n} &= \frac{\pi}{(1 - (1 - \pi)(1 - i))} \cdot \left(\frac{R (1 - i)}{[(1 - i) R + ip_n]}\right)^2 > 0 \\
\frac{\partial CMP}{\partial \theta} &= 0 \\
\frac{\partial CMP}{\partial i} &= \frac{(\pi p_n R)(R - p_n)}{[(1 - i)R + ip_n]^2} \cdot \frac{(1 - i)}{\pi p_n R} \cdot \frac{1}{(1 - (1 - \pi)(1 - i))^2} < 0
\end{align*}
\]

**NAC**

\[
NAC := p_c = 1 - \frac{(1 - \theta)}{\theta} \left(\frac{(p_n - 1)R}{(1 - (1 - \pi)i)R + (1 - \pi)ip_n}\right) \quad \text{(NAC)}
\]

This gives us the following partial derivatives

\[
\begin{align*}
\frac{\partial NAC}{\partial p_n} &= -\frac{(1 - \theta)}{\theta} \frac{(1 - (1 - \pi)i)R^2 + (1 - \pi)iR}{[(1 - (1 - \pi)i)R + (1 - \pi)ip_n]^2} < 0 \\
\frac{\partial NAC}{\partial \theta} &= \frac{1}{\theta^2} \left(\frac{(p_n - 1)R}{(1 - (1 - \pi)i)R + (1 - \pi)ip_n}\right) > 0 \\
\frac{\partial NAC}{\partial i} &= -\frac{(1 - \theta)}{\theta} \left(\frac{(1 - \pi)(R - p_n)(p_n - 1)R}{((1 - (1 - \pi)i)R + (1 - \pi)ip_n)^2}\right) < 0
\end{align*}
\]

Note that \(\frac{\partial NAC}{\partial i} < 0\) since \(p_n \leq R\), otherwise nobody would buy assets in \(t = 1\) and we would end up with a different financial system (i.e. banks hold excess liquidity).
Banks, Markets, and Financial Stability.

**Summary**

\[
\frac{\partial CMP}{\partial p_n} > 0; \quad \frac{\partial CMP}{\partial \theta} = 0; \quad \frac{\partial CMP}{\partial i} < 0; \\
\frac{\partial NAC}{\partial p_n} < 0; \quad \frac{\partial NAC}{\partial \theta} > 0; \quad \frac{\partial NAC}{\partial i} < 0
\]

**Comparative static analysis** \( \theta \)

**Showing that** \( p_n \) **is increasing with** \( \theta \)

Total differential

\[
dp_c = dNAC = \frac{\partial NAC}{\partial p_n} \cdot dp_n + \frac{\partial NAC}{\partial \theta} \cdot d\theta \\
dp_c = dCMP = \frac{\partial CMP}{\partial p_n} \cdot dp_n + \frac{\partial CMP}{\partial \theta} \cdot d\theta
\]

In equilibrium \( dNAC = dCMP \); hence

\[
\frac{\partial NAC}{\partial p_n} \cdot dp_n + \frac{\partial NAC}{\partial \theta} \cdot d\theta = \frac{\partial CMP}{\partial p_n} \cdot dp_n + \frac{\partial CMP}{\partial \theta} \cdot d\theta
\]

Solving for \( dp_n \) and setting \( \frac{\partial CMP}{\partial \theta} = 0 \) leads to

\[
dp_n = \frac{\frac{\partial NAC}{\partial \theta} - \frac{\partial NAC}{\partial p_n}}{\frac{\partial CMP}{\partial p_n} - \frac{\partial NAC}{\partial p_n}} \cdot d\theta
\]

Since

\[
\frac{\partial CMP}{\partial p_n} - \frac{\partial NAC}{\partial p_n} > 0
\]

and

\[
\frac{\partial NAC}{\partial \theta} > 0
\]

we have

\[
\frac{\partial NAC}{\partial \theta} - \frac{\partial NAC}{\partial p_n} > 0.
\]

Consequently \( p_n \) increases with the crisis probability \( \theta \).
**Conclusion**

**Showing that \( p_c \) is increasing with \( \theta \)**

Total differential

\[
dp_c = dNAC = \frac{\partial NAC}{\partial p_n} \cdot dp_n + \frac{\partial NAC}{\partial \theta} \cdot d\theta
\]

\[
dp_c = dCMP = \frac{\partial CMP}{\partial p_n} \cdot dp_n + \frac{\partial CMP}{\partial \theta} \cdot d\theta
\]

Setting \( dp_n = dp_n \), replacing \( \frac{\partial CMP}{\partial \theta} = 0 \) and solving for \( dp_c \) leads to

\[
dp_c = \frac{\frac{\partial CMP}{\partial p_n} \cdot \frac{\partial NAC}{\partial \theta} - \frac{\partial NAC}{\partial p_n} \cdot \frac{\partial CMP}{\partial \theta}}{\frac{\partial CMP}{\partial p_n} \cdot \frac{\partial NAC}{\partial \theta}} \cdot d\theta
\]

It is again straightforward to see that

\[
\frac{\frac{\partial CMP}{\partial p_n} \cdot \frac{\partial NAC}{\partial \theta} - \frac{\partial NAC}{\partial p_n} \cdot \frac{\partial CMP}{\partial \theta}}{\frac{\partial CMP}{\partial p_n} \cdot \frac{\partial NAC}{\partial \theta}} > 0,
\]

hence \( p_c \) increases with the crisis probability \( \theta \).

**Comparative static analysis \( i \)**

**Showing that \( p_c \) is decreasing with \( i \)**

Total differential

\[
dp_c = dNAC = \frac{\partial NAC}{\partial p_n} \cdot dp_n + \frac{\partial NAC}{\partial i} \cdot di
\]

\[
dp_c = dCMP = \frac{\partial CMP}{\partial p_n} \cdot dp_n + \frac{\partial CMP}{\partial i} \cdot di
\]

Gives the solution

\[
dp_c = \frac{\frac{\partial CMP}{\partial p_n} \cdot \frac{\partial NAC}{\partial i} - \frac{\partial NAC}{\partial p_n} \cdot \frac{\partial CMP}{\partial i}}{\frac{\partial CMP}{\partial p_n} \cdot \frac{\partial NAC}{\partial i}} \cdot di
\]

Again the denominator is positive and \( \frac{\partial CMP}{\partial p_n} \cdot \frac{\partial NAC}{\partial i} - \frac{\partial NAC}{\partial p_n} \cdot \frac{\partial CMP}{\partial i} \) is negative. Obviously

\[
-\frac{\partial NAC}{\partial p_n} \cdot \frac{\partial CMP}{\partial i} < 0 \quad \text{and} \quad \frac{\partial CMP}{\partial p_n} \cdot \frac{\partial NAC}{\partial i} < 0 \quad \text{since} \quad \frac{\partial NAC}{\partial i} < 0 \quad \text{and} \quad \frac{\partial CMP}{\partial p_n} > 0.
\]

Thus we conclude \( p_c \) is decreasing with \( i \).
**Ambiguous effect of** \(i\) **on** \(p_n\)

Total differential

\[
dp_c = dNAC = \frac{\partial NAC}{\partial p_n} \cdot dp_n + \frac{\partial NAC}{\partial i} \cdot di
\]

\[
dp_c = dCMP = \frac{\partial CMP}{\partial p_n} \cdot dp_n + \frac{\partial CMP}{\partial i} \cdot di
\]

Solving for \(p_n\)

\[
dp_n = \frac{\frac{\partial NAC}{\partial i} - \frac{\partial CMP}{\partial i}}{\frac{\partial NAC}{\partial p_n} - \frac{\partial CMP}{\partial p_n}} \cdot di
\]

The denominator is positive, but \(-\frac{\partial CMP}{\partial i} > 0\) and \(\frac{\partial NAC}{\partial i} < 0\). Thus the sign of the numerator is ambiguous which translates to the total effect of \(i\) on \(p_n\). Further note that \(\left[\frac{\partial NAC}{\partial i}\right]_{\pi=0} < 0\) while \(\left[\frac{\partial CMP}{\partial i}\right]_{\pi=0} = 0\), hence \(\left[\frac{\partial NAC}{\partial i} - \frac{\partial CMP}{\partial i}\right]_{\pi=0} < 0\). On the contrary, \(\left[\frac{\partial NAC}{\partial i}\right]_{\pi=1} = 0\) while \(\left[\frac{\partial CMP}{\partial i}\right]_{\pi=1} < 0\), hence \(\left[\frac{\partial NAC}{\partial i} - \frac{\partial CMP}{\partial i}\right]_{\pi=1} > 0\). Consequently, the total effect of \(i\) on \(p_n\) depends critically on the fraction of early consuming households \(\pi\).
3 CDS spreads and systemic risk – a spatial econometric approach

Authored by: Armin Eder, Sebastian Keiler

3.1 Motivation

Banks and insurance companies (henceforth financial institutions, FIs, or banks) play a crucial role in an economy by exercising a liquidity transformation, risk transformation and monitoring function. However, the crisis in 2007/2008 illustrated once again that the malfunctioning of the financial system can be disastrous. Being aware of the costly consequences of financial turmoil, regulators and supervisors focused on guaranteeing the soundness of each individual FI by setting minimum capital requirements (e.g. Basel I and II, Solvency I, Swiss Solvency Test) known as micro-prudential regulation and supervision. With the onset of the global liquidity, credit and confidence crisis it became obvious that micro-prudential regulation should be amplified by another dimension, the macro perspective. The need for such an enhancement was underscored by the origins of the financial crisis, where problems in a corner market – the subprime mortgage market, which only accounted for three percent of U.S. financial assets (Eichengreen and Sarno, 2009) – had infected the entire banking system. Eventually this jeopardised the stability of the global financial system. The fact that shocks were transmitted between seemingly independent entities and markets, denoted as contagion or spill over effects, revealed a high degree of financial interconnectedness. And the presence of spill overs across markets and banks indicates a system-inherent mechanism of shock transmission that affects all entities. This contradicts the hypothesis of a system of independent FIs. In the

1 This essay has been featured in Eder, A. and Keiler, S. (2013). CDS spreads and systemic risk – a spatial econometric approach. Bundesbank Discussion Paper, (01/2013). Furthermore, we were able to present our findings on the 4th Central European Regional Science Conference (Bratislava, August 2012) and the Marie Curie Risk Management and Risk Reporting Conference (Konstanz, April 2013). The very helpful feedback is greatly appreciated.
no contagion case, one would expect an institution’s stability to relate solely to its key economic data (the fundamentals). Yet instabilities were quickly transmitted across banks in the recent crisis; we may therefore conclude that it is not only banks’ fundamentals that determine their stability but also the financial system as a whole. It is vital to understand the risks and risk dynamics of a banking system in order to prevent crises.

A bank’s health can be measured by its probability of default, which we measure by its CDS spread. We decompose movements in CDS spreads into a systemic, a systematic and an idiosyncratic risk component by applying a regression approach which originally stems from spatial econometrics. The systematic and idiosyncratic risk factors have a similar interpretation to those used in the capital asset pricing model or the arbitrage pricing model. However, the systemic risk component in our regression is novel to the literature. We set up a parsimonious regression model in which each FI is linked with all the other entities through the financial market. The (single) parameter which indicates the degree of the systemic risk then has to be estimated. This methodology allows to identify systemically important financial institutions and delivers a convenient stress testing tool which combines and interacts macro and micro stress situations.

Contemporaneous CDS spread movements could either result from common exposures or from financial contagion. We explicitly do not characterise movements in CDS markets due to common exposures as contagion and will distinguish between direct effects owing to common exposures and indirect effects which constitute pure contagion. We find the systemic risk charge stemming from financial contagion to be a considerable driver of CDS spreads. Our results underline the importance of the systemic risk component for the probability of default of a financial institution. The risk decomposition furthermore contributes to discussion on the credit spread puzzle.

The next section introduces the relevant literature for our study and seeks to discuss our contributions to the current debate. We then introduce the methodological underpinnings for CDS and the econometric approach, before analysing the data used. In the subsequent section on results, we analyse the findings of our study. The final section summarises and briefly discusses the findings.

3.2 Brief literature overview

This section discusses the theoretical foundations and relevant studies for our analysis. It points out where this study contributes to the current debate. There are numerous theoretical and empirical works on financial contagion. Allen and Gale (2000b) and Freixas...
et al. (2000), for example, explore contagion via the interbank market. Both studies argue that the architecture of the interbank market is of crucial importance and a system where each bank borrows from only one bank is more fragile than a system where the sources of funds are more diversified. In our study we take up the idea of the importance of inter-institutional linkages as a source of risk propagation. However, we do not concentrate solely on bilateral interbank market exposures. In contrast, we analyse contagion stemming from general financial markets. Our model uses common exposures, inferred from equity return correlations, to replicate the financial system.

Another branch of literature makes use of counterfactual simulation methods to analyse the soundness of the interbank market (e.g. Upper 2011; Mistrulli 2011). These studies aim to assess the vulnerability of different interbank market structures to contagious defaults by applying a simulation approach. In general, these models are not well suited to stress testing due to their lack of behavioural foundations. In contrast to counterfactual simulation, the approach applied in this chapter uses regression analysis to explore the effect of financial contagion on market prices in a financial crisis. The advantage of such a model is that it relies on market data being observations of humans’ actions in times of financial turmoil. Therefore, these models are better suited to stress testing and policy analysis.

A further area of theoretical literature argues that financial markets in general may also serve as channels which transmit shocks. In particular, the interaction between funding liquidity and asset liquidity may create systemic risk (e.g. Fecht 2004). The theoretical literature on the transmission of shocks via the asset market has been accompanied by empirical investigations measuring the resilience of the financial system based on market data. That branch of literature defines systemic risk as the $n$th-to-default probability of a portfolio of credit default swaps or an equity portfolio (e.g. Chan-Lau and Gravelle 2005; Lehar 2005). Similarly, Huang et al. (2009) propose an indicator – the price of insurance against large default losses in the banking sector in the coming 12 weeks – to assess the systemic risk of the banking sector. All these empirical studies propose risk indicators which do not shed much light on the shock-transmitting mechanism. Moreover, macro-prudential analysis is entirely detached from the micro perspective in these analyses. In contrast, this analysis directly models the interconnectedness of financial institutions and estimates the degree to which an idiosyncratic shock experienced by one FI is propagated via the financial system.

---

2 $n$th-to-default probability refers to the probability of having $n$ credit events in a CDS portfolio.
In order to capture the phenomenon of financial contagion empirically, it has been modelled econometrically in various ways (Kelejian et al., 2006). One approach is to estimate changes in correlation coefficients of assets after having controlled for the fundamentals. A significant jump in the asset correlation is considered to be evidence of contagion (e.g., Baig and Goldfajn, 1999; Hernández and Valdés, 2001; Fratzscher, 2003). A second approach estimates spill overs in volatility using ARCH or GARCH techniques (e.g., Edwards and Susmel (2000) and Edwards and Susmel (2001)). A third method is to estimate a system of equations where a bank’s probability of default is a function of the probability of default of another bank. This only works if the researcher wants to analyse the spill over effects from a few financial institutions or sectors on others due to an exploding model size (known as the curse of dimensionality). Adams et al. (2010), for example, uses this method. A spatial econometric model, however, is able to overcome the curse of dimensionality and is therefore very well suited to analyse financial data sets. Indeed a spatial model is a restricted version of a system of regressions.

Even though spatial econometric models deliver a straightforward way of modelling the interconnectedness of observations, they have not yet received much attention in finance. One reason for that might lie in the difficulty of finding an adequate measure of economic distance in finance applications. One interesting application of a spatial econometric model in this field has been presented by Fernandez (2011). She develops a spatial capital asset pricing model (S-CAPM) and shows the implications of spatial autocorrelation in asset return series for risk management by deriving a value at risk from the S-CAPM formulation. Spatial econometric models have also been used to model contagion in a macroeconomic context. Kelejian et al. (2006), for example, analyse the extent to which instabilities in the foreign exchange market of one emerging market economy are transmitted to others.

Many other studies have investigated determinants of CDS premiums in the past (see, for example, Alexander and Kaeck (2008) or Ericsson et al. (2009) and extensive comments in Section 3.3.1). Our analysis differs from these studies by additionally modelling the interconnectedness of observations. This allows to measure the degree of financial contagion in the CDS market. Moreover, the model structure allows movements in CDS spreads to be decomposed into a systemic and an FI-specific component, which – to the best of our knowledge – is novel to the literature.
3.3 Methodology

3.3.1 CDS spread dynamics

Credit default swaps offer a hedge against credit risk in which the protection seller offers to compensate the buyer if the underlying defaults before the maturity of the contract. The fee charged by the protection seller, usually on a quarterly basis, is paid up to maturity or up to a previously specified credit event (e.g. the default of the borrower). We denote this fee as the CDS spread and quote it in basis points. Most contracts include a variable upfront payment and no payment is made by the seller if there is no default. The contract design makes a CDS spread an adequate proxy for the probability of default of the borrower. The most striking argument for using CDS to assess a company’s stability is the fact that market data are forward-looking, since today’s price change reflects anticipated future performance changes of the underlying FI. CDS premiums are also considered to be a superior measure of PDs because CDS spreads have been empirically proven to lead bond spreads when responding to shocks. See, for example, Forte and Peña (2009) or Deutsche Bundesbank (2010a).

There are two main approaches to modelling credit risk – the main driver behind CDS spreads. The so called reduced form models focus on modelling an exogenously determined hazard rate which describes the frequency of failure. This approach is used by Jarrow and Turnbull (1995), Jarrow et al. (1997) and Duffie and Singleton (1999) amongst others. A second approach, usually referred to as structural models is based on the contingent claim analysis for valuing debt developed by Black and Scholes (1973) and Merton (1974). This study builds upon the latter. Essentially, this category of models assumes that the default of a firm occurs if the value of its assets drops below a given threshold. While the asset’s value is usually seen as a stochastic process following some trend (i.e. the drift rate, typically approximated using the interest rate level), firm debt is held constant (Ericsson et al. 2009). The original model by Merton (1974) prices risky debt solely as a function of the underlying firm value. Recent studies extend this approach and model credit spreads as a function of firm asset value and other state variables (Collin-Dufresne et al. 2001).

---

3 For an extensive review of relevant variables, see Allen et al. (2004, pp. 124–127)
This section introduces the econometric approach applied in this study. Spatial econometrics, as a branch of econometrics, evolved from the need to incorporate spatial structures within econometric models (Anselin [1988]). In traditional empiric studies the researcher is interested in the functional relationship between a dependent variable (denoted as $y_i$) and several independent variables (denoted as $x_{i,k}$),

$$y_i = f(x_{i,1}, x_{i,2}, \ldots, x_{i,K}) + \varepsilon_i,$$

where $i = \{1, \ldots, N\}$ observations and $k = \{1, \ldots, K\}$ independent variables. $\varepsilon_i$ denotes the error term. For example, in this study we analyse how the CDS spread of company $i$, denoted as $y_i$, depends on that company’s fundamentals $x_{i,k}$ (i.e. financial leverage, equity volatility, etc.). In the classical linear regression $f(.)$ is a linear function. Spatial econometrics advances from the traditional approach by allowing $y_i$ to depend not only on $x_{i,k}$ but also on the dependent variable of all other entities in the system.

$$y_i = f(y_{j\neq i}; x_{i,1}, x_{i,2}, \ldots, x_{i,K}) + \varepsilon_i.$$

By applying such a model structure, we address the question of how the CDS spread of FI $i$ depends on the CDS spreads of all other companies within the financial system. If we find that the CDS spread of company $i$ depends significantly on the CDS spread of any other company, this constitutes evidence of financial contagion. Consequently, such a model allows us to measure and test for the presence of spill over effects. Yet it cannot be estimated in an unrestricted fashion due to the curse of dimensionality.

The research branch of spatial econometrics overcomes this problem with the idea that the information of (geographical) distance between two observations can be exploited. Naturally, first applications stem from the fields of urban and regional economics as well as real estate economics, since distance has a straightforward interpretation in these contexts. In its simplest form, spatial econometric models specify the contiguity of two observations as a binary variable, i.e. being or not being a neighbour. Other studies use measures of geographic and economic distance. Spatial econometrics have quickly found their way into topics such as international economics and labour economics. With the growing attention, a set of estimators, models and tests have been developed that allow various forms of proximity between subjects to be modelled. This study takes up the idea of measuring the effect of economic distance between individuals. It concentrates on
Methodology

financial institutions interacting and working on common markets and reinterprets spatial
spill overs as financial contagion.

Most empirical examinations of the structural approach introduced in Section 3.3.1 regress
a measure of credit risk (in our case CDS spreads) on a set of firm-specific characteristics,
obtaining a classical linear model of the form \( y = X\beta + \varepsilon \). In this simple model, \( y \) is
a specific vector of size \( N \) of observed values of the dependent variable. The matrix of
independent variables is denoted \( X \) and \( \beta \) is the parameter vector. In this framework, \( \varepsilon \)
denotes the usual Gaussian, \( \varepsilon \sim \mathcal{N}(0, \sigma^2 I_N) \), error vector where \( \sigma^2 I_N \) is the covariance
matrix.

In order to capture dependencies across firms within a model framework, the spatial lag
model (or spatial autoregressive model, SAR) introduces the matrix \( W \) by which the
degree of interaction between financial institutions is captured (Anselin, 1988). SAR
models are a sub-category of the general spatial model where the weighting matrix is
introduced to the error term as well. Equation 3.1 specifies the full model and introduces
the term \( \rho Wy \). By the degree of economic proximity determined in \( W \), the CDS spread
of any company \( y_i \) is now dependent on all other CDS spreads \( (y_j) \) in the system and
the parameter \( \rho \) gauges the degree of dependence or, in other words, the intensity of
shock transmission in the system. Note that the element \( i \) of vector \( Wy \) is given by
\[ [Wy]_i = \sum_{j=1,..,N} w_{i,j} y_j \], hence each element of the vector \( Wy \) represents a weighted sum
of the CDS spread of the neighbouring FIs. The spatial autocorrelation coefficient is
bound to \(|\rho| < 1\), for standardised weighting matrices (see Section 3.3.3).

\[
y = \rho Wy + X\beta + \varepsilon
\]

\[
\varepsilon \sim \mathcal{N}(0, \sigma^2 I_N)
\]

(3.1)

The model allows for a decomposition of the CDS premium into a systemic, a systematic
and an idiosyncratic risk component. The systemic risk component reflects the mechan-
ism which transmits shocks via the financial system. The magnitude of risk spill overs
strongly depends on the FI’s interconnectedness and the parameter \( \rho \) measures the degree
of economic dependence. The systematic risk component reflects institutions’ vulner-
ability to changes in general risk factors (e.g. leverage, asset volatility, interest rate, etc.).

The third component, the idiosyncratic risk component (also known as residual risk), is

\footnote{Where, as usual, bold letters represent matrices or vectors and \( I_N \) represents the identity matrix of
size \( N \).}
CDS spreads and systemic risk

the risk to which only the specific FI is vulnerable. In contrast to systemic and systematic risk, idiosyncratic risk can be reduced or eliminated by diversification.

At this point it might be interesting to distinguish between shocks stemming from common exposures and financial contagion. We argue that a statistically significant systemic risk component goes beyond shocks from common exposures which contemporaneously affect institutions’ balance sheets. For example, imagine two FIs have invested in the same firm. A default of that firm will cause both banks to write off parts of their investment. Ceteris paribus, this will decrease equity and increase the leverage of both FIs. Eventually, the change in fundamentals will cause the CDS spreads of both firms to increase. This, of course, is also captured in the classical linear model framework and is reflected as the systematic risk component in the spatial model. However, the existence of a systemic risk component indicates that the total effect of that default is more than just the sum of all direct effects of changing fundamentals. Finding a statistically significant and economically important systemic risk component indicates the presence of a synergy effect arising from banks working in the same financial market. This risk generation process is best summarised in the words of Aristotle, “The whole (the financial system) is more than the sum of its parts (its financial institutions).”

It is particularly interesting to note that the SAR specification nests the classic linear model and a test for the significance of \( \rho \) is a test for the significance of financial contagion. The classical OLS model is not able to capture the effect of financial spill overs. Hence, a statistically significant parameter \( \rho \) not only indicates the relevance of the FI’s interconnectedness to the probability of default of a financial institution; it also implies that standard linear models misestimate the effect of a change in covariates. Thus neglecting the systemic risk component in a regression may lead to biased results by the virtue of omitted variables (LeSage and Pace 2009).

As several authors point out, this category of model differs in interpretation from standard linear models (Anselin 2002). For the classical linear regression model, the effect of a marginal change in any \( x_{i,k} \) is the partial derivative \( \partial y_i / \partial x_{i,k} = \beta_k \). Yet this is not the case for spatial models (Abreu et al. 2004).

\[
y = (I - \rho W)^{-1} X \beta + (I - \rho W)^{-1} \varepsilon \tag{3.2}
\]

---

\(^5\) This holds for all \( i \) and all \( k \), while the partial derivative with respect to every other observation \( j \) is zero: \( \partial y_i / \partial x_{j,k} = 0 \forall i \neq j \).
Equation 3.2 rewrites the SAR model introduced in Equation 3.1. The first part is often denoted as the spatial multiplier: \( S = (I - \rho W)^{-1} \) (LeSage and Pace, 2009). The reader may be familiar with this expression from input/output analysis, where a similar representation is denoted as the Leontief inverse (Anselin, 2003). The nature of spill over effects becomes evident from an in-depth analysis of this term. Since we may rewrite the multiplier as a geometric series \((I - \rho W)^{-1} = I + \rho \cdot W + \rho^2 \cdot W^2 + \rho^3 \cdot W^3 \ldots\), we obtain a system where a shock to any \( y_i \) gets transmitted to any other \( y_j \) forward and backward until it finally diminishes.

In order to separate the contagion from the FI-specific effect, LeSage and Pace (2009) note that the total impact of a change in any \( x_{i,k} \) may be broken down into a direct effect \( \partial y_i / \partial x_{i,k} \) and the indirect effects \( \partial y_j / \partial x_{i,k} \) representing the spill over effects from individual \( i \) on individual \( j \). Since these impacts differ for each individual \( i \), the authors additionally propose a set of summary measures.\footnote{LeSage and Pace (2009) specifically distinguish between the total impact to an observation and from an observation. While these are numerically identical, they give a different interpretation. For the sake of simplicity, this is not discussed any further.}

The average direct impact measures the effect of a change in \( x_{i,k} \) on \( y_i \) – the effect of an individual’s attributes, so to speak. It is computed as the average along the diagonal of the spatial multiplier matrix \( S \) times the parameter vector \( \beta \): \( \Psi_{\text{direct}}^k = \frac{1}{N} \text{tr} (S\beta_k) \). The total effect of a change within the system is denoted as the average total impact. It is calculated as the sum over the \( i \)th row of matrix \( S\beta_k \) yielding the total impact to \( y_i \) from a change in vector \( x_k \). Averaging over all \( N \) sums gives the average total impact: \( \Psi_{\text{total}}^k = \frac{1}{N^i} S\beta_{kt} \), where \( i \) is a column vector of ones of size \( N \). The average indirect impact is obtained by subtracting the direct impact from the total impact: \( \Psi_{\text{indirect}}^k = \Psi_{\text{total}}^k - \Psi_{\text{direct}}^k \).

### 3.3.3 Construction of the weighting matrix

The weighting matrix \( W \) plays a major role within our analysis of financial spill overs. This subsection looks at the construction of this measure of economic distance. The data has a panel structure with \( i = 1 \ldots N \) individuals and \( t = 1 \ldots T \) years consisting of \( s = 1 \ldots S \) months. Interdependence is allowed between companies within the same month (contemporal). Temporal interdependence, i.e. between years and months, is not allowed. Such a system can be obtained via a block diagonal matrix. At the most granular level, we start by constructing correlation matrices from weekly equity returns over a three-year period. Each element within the matrix \( W \) is a correlation coefficient \( \gamma \) of weekly stock returns between two financial institutions based on a three-year period \( w_{i,j} \equiv \gamma_{i,j} \forall i \neq j \) in year \( t \) and month \( s \) and 0 else – this models contemporaneous economic distance only.
CDS spreads and systemic risk

In order to obtain exogeneity of the weighting matrix, we lag the correlation coefficient used as a weight \( w_{i,j} \) by one year.\(^7\)

We standardise the weighting matrix by dividing each element of the matrix by the maximum absolute eigenvalue of \( W \). This ensures that the spatial model will be stationary if \( \rho \in (\min(\eta)^{-1}, 1) \), where \( \eta \) is a vector of size \( n \) of eigenvalues of the matrix \( W \). Furthermore, it retains the absolute distance.\(^8\)

For clarification, in Equation 3.3 the described structure is reinterpreted in sum-notation. It builds upon Equation 3.1 the SAR model, for month \( s \) and year \( t \). The focus, again, lies on the lagged weighting element \( w^*_{i,j,s,t-1} \), which is the lagged correlation coefficient divided by the maximum absolute eigenvalue. This obtains a standardised weighting matrix for all \( i \neq j \) The main diagonal elements are zero by definition.

\[
y_{i,s,t} = \rho \cdot \sum_{j=1, i \neq j}^{N} (y_{j,s,t} \cdot w^*_{i,j,s,t-1}) + \sum_{k=1}^{K} (\beta_k \cdot x_{i,k,s,t}) + \varepsilon_{i,s,t} \tag{3.3}
\]

Element \( y_{i,s,t} \) – the CDS spread of FI \( i \) in month \( s \) of year \( t \) – is modelled as a weighted average of the scaled correlation coefficients \( w^*_{i,j,s,t-1} \) times the CDS spread of other companies in month \( s \) of year \( t \) \( (y_{j,s,t}) \) multiplied by the spatial autocorrelation parameter \( \rho \) (estimated over all periods). The exogenous determinants of the FI, the systematic risk, is added to this systemic component by including the economic situation and firm fundamentals in our regression (the second sum over all risk factors \( K \)). The Gaussian error term \( \varepsilon_{i,s,t} \) represents the unsystematic risk.

However, the use of equity correlations as a measure of economic distance could be problematic if shocks influence equity correlations as well as CDS spreads. In such a case, the measure of economic distance was endogenous and statistical properties of estimators do not hold anymore (LeSage and Pace, 2009). In order to circumvent this issue, the weighting matrix is constructed by lagging the correlations by twelve months. Nevertheless, this was still insufficient if the contemporary CDS spreads \( y_{i,s,t} \) contained information for

\(^7\) An extensive treatment on the construction of the weighting matrix can be obtained from the authors upon request.

\(^8\) Note that \( \min(\eta)^{-1} = -6.2 \) in our analysis.

\(^9\) Spatial econometricians typically row standardise the weighting matrix (Plümper and Neumayer, 2010). This means that each matrix cell is divided by its row sum, such that the weights in each row add up to one. The absolute distance is thus altered to a relative measure of distance. Since row standardisation implies that the spill over effect on each bank in the financial system decreases proportionally with the degree of interconnectedness of the bank which initially faced the shock. Such an assumption is problematic in our analysis. See Plümper and Neumayer (2010) for a detailed discussion of row standardisation.
explaining the correlation of equity returns \( w_{i,j,s,(t-1)} \), used for constructing the weighting matrix. In our case, the correlation of the average CDS spread and the average logarithmic return in \((t - 12)\) is not present \((\gamma = -0.0001)\), indicating (at least) no linear dependence of equity returns in \((t - 1)\) and CDS spreads in time \(t\). We conclude from this that the matrix \( W \) can be treated as exogenous.

In general, the construction of the weighting matrix has been the subject of heated debates. Other variants have been proposed (see for example Fernandez, 2011). However, the results reported in Section 3.5 are robust to changes within \( W \).

Table 3.1 summarises the correlations in matrix \( W_t \). On average, moderate correlations of roughly 0.7 predominate for all years. Compared to the mean, the median is slightly shifted towards 1. The average correlation slightly decreases during 2003–2007. In the crisis years, average correlations rise again. This is a common pattern for stocks during periods of financial distress.

<table>
<thead>
<tr>
<th>Year</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>0.66</td>
<td>0.26</td>
<td>0.74</td>
<td>0.00</td>
<td>0.98</td>
</tr>
<tr>
<td>2004</td>
<td>0.70</td>
<td>0.23</td>
<td>0.79</td>
<td>0.00</td>
<td>0.96</td>
</tr>
<tr>
<td>2005</td>
<td>0.71</td>
<td>0.22</td>
<td>0.78</td>
<td>0.04</td>
<td>0.96</td>
</tr>
<tr>
<td>2006</td>
<td>0.69</td>
<td>0.19</td>
<td>0.71</td>
<td>0.08</td>
<td>0.96</td>
</tr>
<tr>
<td>2007</td>
<td>0.53</td>
<td>0.24</td>
<td>0.57</td>
<td>0.01</td>
<td>0.88</td>
</tr>
<tr>
<td>2008</td>
<td>0.71</td>
<td>0.28</td>
<td>0.87</td>
<td>0.08</td>
<td>0.98</td>
</tr>
<tr>
<td>2009</td>
<td>0.72</td>
<td>0.27</td>
<td>0.84</td>
<td>0.09</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Table 3.1: Descriptive statistics for the CDS weighting matrix, broken down by years. Note that only the correlations between 2003 and 2008 have been used for estimation. Furthermore, the main-diagonal elements have been excluded.

However, standard deviations show that an increased heterogeneity has been observed during the crisis. High return correlations indicate economic proximity and thus a high likelihood of financial contagion. A closer inspection of patterns leads to the conclusion that European and American financial institutions in particular are highly interconnected. This observation does not hold for Japanese FIs. While US and European institutions have average correlations of 0.73 and 0.69 respectively (see Table 3.2), Japanese institutions’ equity correlation is 0.49 (calculated over the whole sample and time span). Therefore, the Japanese financial system is not as prone to contagion spreading from other continents as the European or American one. This has also been reported by the Deutsche Bundesbank (2010b) when investigating the German financial system’s susceptibility to

\(^{10}\) For more details about the sensitivity of reported results with respect to changes within \( W \), see Appendix C
financial shocks. This particular study showed that financial disruptions in Europe or in the U.S. will strongly affect the soundness of the German financial system, while Japanese financial shocks have no significant impacts. This is in line with the architecture of $W$. From the table it is also evident that the interconnectedness, and thus the expected spill over effects, will be highest for European institutions, followed by American companies.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>EU</td>
<td>0.73</td>
<td>0.22</td>
<td>0.80</td>
<td>−0.20</td>
<td>0.98</td>
</tr>
<tr>
<td>JP</td>
<td>0.49</td>
<td>0.27</td>
<td>0.48</td>
<td>−0.20</td>
<td>0.98</td>
</tr>
<tr>
<td>USA</td>
<td>0.69</td>
<td>0.23</td>
<td>0.75</td>
<td>−0.18</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Table 3.2: Descriptive statistics for the CDS weighting matrix, broken down by regions. Calculated as the average correlation of all entities within a region over the whole sample and time span.

### 3.4 Data

As initially described, we base our analysis on five-year credit default swap spread data obtained from Bloomberg. The choice of financial institutions is based on a preliminary list of systemically important financial institutions announced by the Financial Stability Board in 2010 ([www.financialstabilityboard.org](http://www.financialstabilityboard.org)). In this study we use a balanced panel of 15 out of the originally 30 institutions. This restriction has been necessary since data for the chosen period of time has not been available for all 30 entities. Given the alternatives of a full set but unbalanced panel of institutions and a balanced panel over a long time span the latter alternative has been chosen for statistical reasons. It is governed by statistical properties of the used estimators ([LeSage and Pace](2009)).

A full list of all institutions along with their abbreviations is provided in the appendix. Our data covers the 2004–2009 period at a monthly frequency. To ensure comparability, all data have been calculated as end-of-month data. **Figure 3.1** depicts the median CDS spread over the whole sample period. The grey area indicates the .75 and the .25 percentiles. The impact of the market turmoil in 2008 and the turbulence in 2009 are evident. This is also reflected in the key figures displayed in **Table 3.3** where the average CDS spread rises from 22.32 in 2004 to 121.35 in 2008.

For exploring CDS spreads, **Section 3.3.1** has already outlined the main theoretical determinants. They largely depend on a firm’s asset value, its asset volatility and some other macro variables. The data were obtained from Bloomberg. Since financial institutions
Table 3.3: Descriptive statistics for the CDS spreads in the sample.

<table>
<thead>
<tr>
<th>Year</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>22.32</td>
<td>7.74</td>
<td>20.53</td>
<td>9.67</td>
<td>47.10</td>
</tr>
<tr>
<td>2005</td>
<td>18.24</td>
<td>6.02</td>
<td>17.78</td>
<td>7.70</td>
<td>35.19</td>
</tr>
<tr>
<td>2007</td>
<td>25.48</td>
<td>18.99</td>
<td>19.75</td>
<td>4.62</td>
<td>97.20</td>
</tr>
<tr>
<td>2008</td>
<td>121.35</td>
<td>96.80</td>
<td>101.93</td>
<td>42.50</td>
<td>1033.50</td>
</tr>
<tr>
<td>2009</td>
<td>136.50</td>
<td>98.89</td>
<td>105.83</td>
<td>43.33</td>
<td>631.53</td>
</tr>
</tbody>
</table>

from three different geographical regions (Asia, Europe and USA) and five currencies are covered (CHF, EUR, GBP, USD and JPY), we match FIs with their region-specific variables for the respective currency area, such as the interest rate. Variables quoted in a foreign currency are converted to USD using the appropriate exchange rate. We construct monthly time series for fundamental data by interpolating the available quarterly or semi-annual time series with a cubic spline. However, the results are neither sensitive to the method of interpolation (e.g. linear, or by carrying forward the last observation) nor to the chosen frequency. As a result, our findings remain qualitatively the same for a quarterly frequency or other interpolation methods. The following paragraphs contain an in-depth discussion of the regressors.

**Leverage:** The leverage describes the ratio between debt plus equity and debt. Since greater leverage leaves a higher equity cushion in the case of economic distress, greater leverage is connected with a higher probability of default (Ericsson et al., 2009; Merton, 1974). This suggests a positive relationship between this variable and the CDS spread. Moreover, the leverage is closely related to the recovery rate – the
CDS spreads and systemic risk

part of the loan that can be recovered in the event of a default. Higher leverage decreases the recovery rate. The recovery rate is negatively related to the CDS spread (Deutsche Bundesbank, 2010a). The interaction between leverage and the recovery rate theoretically amplifies the impact on the explained variable. In this analysis, the leverage is measured as the ratio of total liabilities over total liabilities plus market capitalisation.\footnote{The chosen definition differs from the Definition adapted in Basel II. For comparability it is, however, consistent with the chosen definition in Ericsson et al. (2009).}

**Firm asset volatility:** Higher asset volatility implies a greater probability of the firm’s asset value falling below the threshold. In Merton’s Model, a firm’s debt is equivalent to holding a risk-free bond plus a short put on the firm’s equity. From option pricing theory it is evident that a higher volatility implies a higher price for the put. Both arguments suggest a positive relationship between asset volatility and CDS spreads. Firm asset volatility is proxied by a GARCH(1,1) fit of daily stock returns, converted to end-of-month data in order to fit the frequency used in the regression.

**Interest rates:** Interest rates play a crucial role in the assessment of credit risk. The term structure of interest rates condenses information on the economic condition. Three common measures summarise the yield curve. The level of the yield curve describes the general interest rate level in the economy and serves as a proxy for the drift rate of the firm’s assets. From the Merton model alone – and neglecting any effect of the yield level on refinancing costs –, we expect an inverse relationship of the interest rate level with CDS spreads since a higher drift rate implies a smaller probability of default. The steepness of the yield curve summarises the relative cheapness of short-term debt compared to long-term debt. On an insurance company’s balance sheet, for example, long-term liabilities are financed with shorter-term assets. For Germany, a flat yield curve is currently observable this poses pressure on insurers (Wilson, 2012). Contrarily, banks’ assets typically exceed the duration of their liabilities. Such asset-liability-duration mismatches, combined with a change of the yield curve shape, might deteriorate FI’s solvency. Finally, the curvature may point towards a hump-shaped yield curve. Hump-shaped yield curves indicate economic disruptions (Haubrich and Dombrosky, 1996). In our model, the level of the yield curve is represented by the 10-year spot rate for each geographical region. The difference between the 10-year yield and the three-month yield serves as an estimate
Data

for the slope of the curve. Finally, the curvature is estimated by $2 \cdot r_{12m} - (r_{10yr} + r_{3m})$ (Ericsson et al., 2009).  

Credit rating: The credit rating review process relies not only on public but also on non-public information. Thus, in addition to market data, ratings and rating events may explain the level of and changes in the CDS spread respectively. We expect an upgrade to have a significantly negative impact on the CDS spread. For downgrades an inverse reaction should take place. We use credit rating information from Standard & Poors.

Liquidity: In an imperfect market environment, liquidity is a determinant of CDS spreads. A liquidity premium has been identified by previous studies (Deutsche Bundesbank, 2004; Longstaff and Mithal, 2005). It is therefore reasonable to expect a higher CDS spread for periods of illiquidity within a market. The bid-ask spread, averaged over all FIs in period $t$, is included in order to account for changes in market liquidity.

Firm size: The onset of the market turmoil in 2008 highlighted the fact that some institutions are systemically important. Thus, these institutions are more likely to be saved from default (Völz and Wedow, 2009). Anticipating this behaviour, firm size might have a negative impact on the dependent variable. Furthermore, firm size may serve as a proxy for the degree portfolio of diversification and thus bigger firms might be less risky (Basel Committee on Banking Supervision, 2006). Another feature of firm size may be given by economies of scale (Elsas and Hackethal, 2010). All these effects suggest a negative relationship between firm size and the probability of default measured as the CDS spread. The size of a company is represented by the natural log of total assets.

Abnormal return: The residuals of a regression of the equity return adjusted for the risk-free rate on the adjusted market return should capture the FI’s exceptional performance. This measure is thus simply the residual of a CAPM regression.

Economic state variables: Along with a change in fundamentals, the business climate may also affect the probability of default. Evidence on the state of the economy can be provided by the spread between the London Interbank Offered Rate (Libor) and the Overnight Indexed Swap rate (OIS), capturing the cost of credit risk as well as the health of the banking system. The Libor-OIS spread is the difference between the interest rate at which banks are willing to grant loans to other banks for

12 While other measures for yield curve characteristics are theoretically possible (e.g. the first three principal components or parameter estimates of a Nelson-Siegel model), the results are robust to variations in the construction of this measure.
**CDS spreads and systemic risk**

a pre-specified term and the overnight funds rate expected by the markets. While the Libor market is risky, the OIS market is almost default-free because there is no exchange of principals – counterparties swap the fixed rate for the floating rate – and payments are made only at maturity (Thornton 2009). Consequently, the Libor-OIS spread reveals information on the soundness of the banking system or, as Alan Greenspan put it, “Libor-OIS remains a barometer of fears of banks insolvency” 13. Since a widening of this spread is associated with economic distress, we suggest a positive relationship.

### 3.5 Results

In this section, we report the results for the 15 examined FIs using the models introduced in Section 3.3.2. First, we present the estimation results for both CDS premium levels and changes in CDS premiums. Subsequently, we illustrate FIs’ susceptibility to financial infection by stress testing individual institutions.

#### 3.5.1 Investigating CDS spreads

As outlined in Section 3.3.1, structural models, introduced by Merton (1974), identify three risk drivers for pricing risky debt: the risk-free rate, the firms’ asset volatility and the firms’ capital structure. While the insight offered by this theory is fundamental, it fails to empirically explain the complex credit spread dynamics (Kim et al. 1993). Responding to this issue, we report results for a restricted set of regressors along the lines of Merton (1974) as well as a fully specified set of explanatory variables introduced by various other authors. Moreover, both models are estimated taking into account spatial dependence (sar and dsar) and without spatial dependence obtaining a classic linear regression (ols and dols). Variants of the latter model were mainly used in literature for assessing determinants of CDS spreads in the past. All models are estimated with individual fixed effects by means of maximum likelihood. Table 3.4 compares regression results for absolute CDS spreads (levels denoted ols and sar) and first differences (changes

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13 This quote has been taken from an interview given by Alan Greenspan in 2009 (Finch and Mc-Cormick 2009).
in CDS spreads labelled *dols* and *dsar*.\(^\text{14}\) All models have been estimated as a pooled regression and no time lag structures have been introduced to the specifications.\(^\text{15}\)

\(^{14}\) We also examined models with further explanatory variables capturing the banks’ health and profitability or the state of the economic and financial system, e.g. GDP growth, market portfolio return, inflation, return on equity, etc. However, none of these variables contributed to model performance measured by *AIC* or the *p*-value of the regressor.

\(^{15}\) With such a model, the time structure is not explicitly taken into account.
<table>
<thead>
<tr>
<th>Variable</th>
<th>ls1</th>
<th>sar1</th>
<th>ls2</th>
<th>sar2</th>
<th>Variable</th>
<th>dls1</th>
<th>dsar1</th>
<th>dls2</th>
<th>dsar2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>-10.36***</td>
<td>-5.86*</td>
<td>-5.46</td>
<td>-5.42</td>
<td>\Delta Level</td>
<td>-27.06***</td>
<td>-15.21**</td>
<td>-4.29</td>
<td>-2.39</td>
</tr>
<tr>
<td>Slope</td>
<td>5.91</td>
<td>5.33</td>
<td>(2.99)</td>
<td>(2.83)</td>
<td>\Delta Slope</td>
<td>-21.04*</td>
<td>-20.64*</td>
<td>(8.33)</td>
<td>(8.18)</td>
</tr>
<tr>
<td>Curvature</td>
<td>14.01***</td>
<td>13.99***</td>
<td>(3.49)</td>
<td>(3.43)</td>
<td>\Delta Curvature</td>
<td>2.14</td>
<td>1.07</td>
<td>(7.35)</td>
<td>(7.18)</td>
</tr>
<tr>
<td>(\sigma_{Equity})</td>
<td>14.10***</td>
<td>9.92***</td>
<td>4.48***</td>
<td>4.55***</td>
<td>\Delta (\sigma_{Equity})</td>
<td>2.91***</td>
<td>2.30***</td>
<td>1.13*</td>
<td>1.23*</td>
</tr>
<tr>
<td>log(assets)</td>
<td>-33.78***</td>
<td>-35.20***</td>
<td>(5.96)</td>
<td>(5.87)</td>
<td>\Delta log(assets)</td>
<td>-15.72</td>
<td>-6.52</td>
<td>(22.39)</td>
<td>(21.89)</td>
</tr>
<tr>
<td>Leverage</td>
<td>12.55***</td>
<td>8.10***</td>
<td>6.63***</td>
<td>6.65***</td>
<td>\Delta Leverage</td>
<td>3.89</td>
<td>0.43</td>
<td>(2.84)</td>
<td>(2.56)</td>
</tr>
<tr>
<td>Libor-OIS spread</td>
<td>52.85***</td>
<td>51.40***</td>
<td>(0.79)</td>
<td>(0.76)</td>
<td>\Delta Libor-OIS spread</td>
<td>16.44*</td>
<td>16.07*</td>
<td>(8.18)</td>
<td>(8.11)</td>
</tr>
<tr>
<td>Bid-ask spread</td>
<td>6.14***</td>
<td>4.42***</td>
<td>(0.67)</td>
<td>(0.82)</td>
<td>\Delta Bid-ask spread</td>
<td>9.47***</td>
<td>7.25***</td>
<td>(0.65)</td>
<td>(0.76)</td>
</tr>
<tr>
<td>Rating A–</td>
<td>24.91***</td>
<td>22.58***</td>
<td>(6.35)</td>
<td>(6.27)</td>
<td>Downgrade</td>
<td>24.88**</td>
<td>20.52*</td>
<td>(0.14)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>Rating AA</td>
<td>-58.85***</td>
<td>-56.78***</td>
<td>(8.05)</td>
<td>(7.95)</td>
<td>Abnormal Return</td>
<td>-1.08***</td>
<td>-1.08***</td>
<td>(9.37)</td>
<td>(9.16)</td>
</tr>
<tr>
<td>Rating AA–</td>
<td>-50.08***</td>
<td>-46.76***</td>
<td>(6.23)</td>
<td>(6.24)</td>
<td>(\rho)</td>
<td>0.55***</td>
<td>0.24***</td>
<td>(0.03)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>(\rho)</td>
<td>0.40***</td>
<td>0.13**</td>
<td>(0.03)</td>
<td>(0.04)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Obs             | 1,080 | 1,080 | 1,080 | 1,080 | Obs             | 1,065 | 1,065 | 1,065 | 1,065 |
| adj. \(R^2\)   | 0.65  | 0.69  | 0.75  | 0.74  | adj. \(R^2\)   | 0.02  | 0.19  | 0.28  | 0.31  |
| AIC             | 11,332 | 11,178 | 10,991 | 10,986 | AIC             | 10,862 | 10,704 | 10,541 | 10,527 |
| BIC             | 11,426 | 11,278 | 11,131 | 11,131 | BIC             | 10,956 | 10,804 | 10,675 | 10,666 |

Table 3.4: Determinants of CDS-Premiums; standard errors in parentheses; * significant at the 5% level; ** significant at the 1% level; *** significant at the 0.1% level.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Regression in levels</th>
<th>Regression in differences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sar2 direct</td>
<td>sar2 indirect</td>
</tr>
<tr>
<td>Level</td>
<td>−5.43</td>
<td>−0.69</td>
</tr>
<tr>
<td>Slope</td>
<td>5.33</td>
<td>0.68</td>
</tr>
<tr>
<td>Curvature</td>
<td>14</td>
<td>1.78</td>
</tr>
<tr>
<td>σEquity</td>
<td>4.55</td>
<td>0.58</td>
</tr>
<tr>
<td>log(assets)</td>
<td>−35.24</td>
<td>−4.49</td>
</tr>
<tr>
<td>Leverage</td>
<td>6.66</td>
<td>0.85</td>
</tr>
<tr>
<td>Libor-OIS spread</td>
<td>51.46</td>
<td>6.55</td>
</tr>
<tr>
<td>Bid-ask spread</td>
<td>4.43</td>
<td>0.56</td>
</tr>
<tr>
<td>Rating A+</td>
<td>−36.17</td>
<td>−4.6</td>
</tr>
<tr>
<td>Rating A−</td>
<td>22.61</td>
<td>2.88</td>
</tr>
<tr>
<td>Rating AA</td>
<td>−56.85</td>
<td>−7.24</td>
</tr>
<tr>
<td>Rating AA−</td>
<td>−46.81</td>
<td>−5.96</td>
</tr>
</tbody>
</table>

Table 3.5: Direct and Indirect Effects.
The table presents least squares estimation results for the Merton model (ls1 and dls1) and for a fully specified set of regressors (ls2 and dls2). The results denoted sar1 and dsar1 include the proposed methodology of introducing a spatial lag for the restricted set of regressors as well as for all regressors (sar2 and dsar2).

Obviously, the risk factors in the Merton model play a statistically significant and economically important role in CDS spreads. Yet the estimated parameters for restricted models are higher throughout. To be more precise, the restricted model versions lead to parameter estimates being twice (yield level or leverage) or even three times (equity volatility) as high as the models with the full set of explanatory variables. This result is even more pronounced for the changes in CDS spreads. Such a pattern may point to an omitted variable bias for the restricted set of regressors. In the absence of further explanatory variables, the remaining variables explain variation in \( y \) (the CDS spread in absolute terms and CDS spread changes) that pertains to omitted variables. This is reflected in a higher estimate for the coefficients.

Introducing the spatial lag strongly improves model quality measured in terms of the Akaike Information Criterion (AIC). The striking decrease in the AIC when estimating the ls1 model with a systemic risk factor (sar1) suggests that the spatial lag potentially captures the effect of omitted explanatory variables. In general, if important variables are omitted from a spatial regression, the spatial lag and the other explanatory variables will capture their effects. In contrast, introducing the spatial structure when a full set of regressors is present (ls2 and dls2) only moderately improves the model fit (see column sar2 as well as dsar2). However, the spatial lag remains highly significant. We interpret this as evidence for the lack of omitted relevant variables. In such a case, the spatial lag purely measures the importance of financial spill overs.

The estimation result provides strong evidence that it is not only an FI’s individual fundamentals and the macro-economic conditions that matter when assessing its solvency; the risk of financial contagion as modelled by the spatial econometric approach is highly important as well. We argue in Section 3.3.2 that a test for the significance of the spatial autocorrelation coefficient (\( \rho \)) is a formal and straightforward way of testing for the presence of financial contagion. Indeed, we find the spatial lag to be highly significant; consequently, risk propagation plays an important role in the CDS market. Additionally, the test reveals that models which do not take financial spill overs into account – such as

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16 This also coincides with [LeSage and Pace (2009)](https://link), who reinterpret spatial models in a missing variable context along with three further motivations for applying this category of model. A similar rationale also exists for time series models.
the classical linear regression – are most probably misspecified. Such models might yield biased results (LeSage and Pace, 2009).\(^{17}\)

Despite the significance of contagion effects, the analysis of the magnitude is only made feasible by splitting the total effect of a financial shock into a direct and an indirect component. The spatial econometric approach allows for the separation of the direct effects of a change in bank A’s fundamentals on bank A’s solvency as well as the indirect impact due to spill over effects on the solvency of bank B – and all other banks – caused by the same change. Table 3.5 summarises the average total, direct and indirect effect of each risk driver in the SAR models and compares the total effect with that of ordinary least squares regressions (ls2 and dls2).

For CDS spread levels, it is particularly interesting to see that more than a tenth of the total effect of a shock is due to financial spill overs. This again highlights the economic relevance of financial spill overs in the CDS market. The result can also be interpreted as strong evidence for the need for macroprudential supervision and regulation. Our regression portray the financial system’s role for banks. In a model assuming \( N \) independent financial institutions (such as in an OLS regression), the effect of financial stress was reduced by 10 per cent on average. It is also important to note that not only the stressed FI affects the solvency of other banks; there is also a feedback effect stemming from the fact that the reduced solvency of the sound bank causes stress to the bank which initially faced the shock. This is an inherent feature of the spatial multiplier matrix and it shows how upward and downward spirals may evolve (See Section 3.3.2 for an explanation of the mathematical methodology). In good times, positive contagion drives the expected default rate down to zero. In periods of financial distress, however, spill over effects lead to soaring credit spreads for all FIs in the financial system. This causes the health of the entire economic system to deteriorate. From this perspective, the results should prompt macroprudential regulators to develop a regulatory framework which is able to prevent contagion among banks.

When comparing the total effect of the sar2 model with the parameter estimates of the ls2 model, it is striking that effects in the sar2 model exceed the linear model parameter estimates. Such an effect may originate from the misspecification of the classical linear model. Furthermore, the effects in Table 3.5 indicate that for monthly changes in CDS spreads (dsar2) more than a fifth of the total effect is due to financial contagion. This is a substantial increase compared to the figure of 10 per cent reported for the regression in levels. The latter may perhaps be just the lower bound. Thus, financial spill overs possibly

\(^{17}\) For more details on potential misspecification of the SAR model, see Appendix B.
account for more than 10 per cent of the total variation in CDS premiums, stressing the conclusions we draw from analysing CDS spread levels.

Regarding the systematic risk component our findings are compatible with the existing literature. The risk factors suggested from the Merton model alone turn out to be insufficient to explain credit risk, yet they are highly relevant for explaining credit spreads. In line with other studies, the effect of interest rates on credit spreads in general delivers ambiguous results. For example, [Alexander and Kaeck (2008)] find that the yield level has a significant and negative effect on credit spreads in all examined sectors but the financial sector. Additionally, [Deutsche Bundesbank (2011)] find German small and medium sized banks to be sensitive to changes of the yield curve, while big financial institutions are hedged very well. After all the influence of the yield environment highly depends on the individual asset liability structure of the respective FI. Furthermore, the market’s liquidity – measured by the bid-ask spread – and firms’ credit rating turn out to be crucial determinants of credit spreads. According to our findings, the average health of the financial system, measured as the Libor-OIS spread, and firms’ extraordinary performance, measured by the excess return over the CAPM model, are also a relevant risk factor in the CDS market. Finally, we do find evidence that the market rewards firm size with a discount. This may support the conclusion that (a) bigger companies are more likely to be rescued, or (b) are in a better position to diversify risk in their portfolios, or (c) are more profitable due to economies of scale.

### 3.5.2 Stress testing the financial system

In this subsection we provide an in-depth analysis of the results obtained from the regressions presented above. We demonstrate the mechanism at work as well as the implications of financial contagion based on scenario analysis. To examine the financial system, we apply a shock to certain entities and perform predictions based on estimation results. Recall that the main difference between a spatial model and the classical linear model is the assumption about the interconnectedness of FIs. Implicitly linear regressions assume independence across the system. In other words, the solvency of an entity is solely a function of its fundamentals; consequently, an FI’s solvency will not be affected by a change in the fundamentals of another individual in the financial system.

On 15 September 2008 Lehman Brothers Inc. filed for bankruptcy, causing financial turmoil around the world. In our sample, for instance, this more than doubled end-of-month equity volatility compared to the previous month. We use economic conditions in September 2008 as our baseline scenario for assessing the outcome of an economic shock
in December 2009. For this purpose, all fundamental variables \((x_{i,k,Dec2009})\) are shocked in such a way that we end up with the institution’s fundamentals in 2008 \((\Delta x_{i,k} = x_{i,k,Sep2008} - x_{i,k,Dec2009} \forall k = 1 \ldots K \text{ and } i = 1 \ldots N)\). However, the interconnectedness is modelled using the 2009 weights. We compare predicted CDS spreads during September 2008 conditions with today’s CDS markets.\(^{18}\)

Only European banks are stressed in our first scenario (see Figure 3.2). We carry out predictions \((\hat{y}_{i,k})\) for the left-hand variable in December 2009 using the in-sample values for \(x_{i,k} \forall i \in \text{Non-Europe}\) as well as the altered variables \(x_{j,k,Dec2009} + \Delta x_{j,k} \forall j \in \text{Europe}\). The left-hand panel (a) of Figure 3.2 depicts the predicted changes in CDS spreads for the OLS model. In contrast, the right-hand panel (b) shows the predictions from the SAR model. Since OLS assumes \(N\) independent financial institutions, only the European banks and insurance companies are affected. In contrast to the OLS model, the SAR model also predicts increasing spreads for all other companies (panel (b)) due to financial contagion. When comparing the magnitude of the spill over effects, it is striking that American FIs suffer more from contagion than Asian ones. This is especially true of Sumitomo Mitsui Financial Group (SUMIBK) because of its low financial interconnectedness (we already highlighted the lower connectivity between Asian and European institutions in Section 3.3.3).

Our first scenario explored the differences between a classical linear model and the proposed SAR-category models. The second scenario explores the mechanism further by shocking only the equity volatility (see Figure 3.3). Here, the emphasis lies on the decomposition of the total effect into direct and indirect effects. By triggering a jump in the equity volatility alone, we introduce a state of increased uncertainty (left-hand panel (a)). The decomposition highlights the fact that not every FI suffers from contagion (i.e. the indirect effect) to the same extent. A comparison of Banco Bilbao Vizcaya Argentaria S.A. (BBVASM) and SUMIBK illustrates the ambivalent nature of systemic risk. On one hand, BBVASM faces almost no increase of its equity volatility in our scenario. The direct effect hardly increases CDS premiums. SUMIBK, on the other hand, faces a substantial increase in equity volatility that drives CDS charges substantially upward. When systemic connectivity comes into play a very different pattern evolves. BBVASM suffers from its high connectivity to other institutions (e.g. Citi Inc., Morgan Stanley

\(^{18}\) For the comparative static analysis, we used the ols2 and sar2 model with the first differences of CDS premiums as the dependent variable.

\(^{18}\) The x-axis plots CDS changes in basis points and the y-axis lists the financial institutions in the sample; each plot shows the impact of the reoccurrence of the September 2008 scenario on the CDS premiums observed in December 2009 \((\Delta x_{i,k} = x_{i,k,Sep2008} - x_{i,k,Dec2009})\).
Figure 3.2: Outcomes from stress testing the financial system in scenario 1. (a) shows the expected change in the CDS premiums forecasted by the linear model due to a stress to European FIs only. (b) is a forecast of the spatial model of the same shock.

The third scenario features a fully unrestricted change in variables for all entities in the sample (see Figure 3.3). Now the whole change in fundamental and economic variables is applied to the SAR. Interestingly, the direct effect of that shock causes the CDS spread of Mizuho Financial Group (MIZUHOB) to decrease slightly whereas all other institutions face a severe increase in their CDS spreads. Thus, the enhanced solvency is accompanied by a deteriorating financial environment in this scenario. Since the institution is part of the financial system and suffers from financial contagion, the indirect effect also causes its CDS spread to increase in total.

The proportion of the indirect effect relative to the direct impact reveals not only banks’ vulnerability to financial contagion but also draws the attention to a broadly discussed phenomenon known as the credit spread puzzle (Tsuij 2005). In a nutshell, this term describes the non-linear relationship between expected loss (EL) and observed credit spreads. For small expected losses, there is a wide gap between the observed credit spread and the EL. This is not typically true of higher ELs. Panel (b) in Figure 3.3 illustrates this relationship.
Figure 3.3: Outcomes from stress testing the financial system in scenario 2 and 3. (a) shows the impact of a jump in the equity volatility of each FI broken down into a direct effect (black) and an indirect effect (grey). (b) Direct and indirect effects from employing the shock to each variable to the whole financial system.

3.6 Conclusion

This study explicitly models the degree of proximity between financial institutions, approximated by the equity correlation between two firms. The framework of spatial econometrics offers an efficient way of introducing banks’ interconnectedness in terms of equity correlation into a regression model. Moreover, the spatial econometric approach allows for a decomposition of the variance of banks’ CDS premiums into a systemic, a systematic and an idiosyncratic risk component. Our results indicate that the systemic risk component in the CDS market is important. The extent of spill over effects – measured

relationship: For entities with a smaller change in the credit spread, the systemic charge (i.e. indirect effect) is relatively larger than for FIs with a considerable change in the CDS premium. The risk propagation mechanism offers the explanation that the credit spread puzzle may originally evolve from systemic risk. Since this kind of risk is not diversifiable (Tsuji 2005), markets demand not only an individual credit risk charge but also a systemic risk charge.
CDS spreads and systemic risk

by the spatial autoregressive parameter – and the magnitude of transmission summarised as direct as well as indirect effects are found to be significant and considerable. These effects are still present even when macroeconomic variables such as the Libor-OIS spread or proxies for risk mitigation are introduced into the model. We find that the systemic risk charge varies by geographical region as well as by time. While European and US institutions are strongly affected by financial contagion, Asian banks are found to be rather independent. Overall, we find CDS spreads to be up to a tenth higher than a model without spill over effects would predict. For first differences of CDS spreads, the degree of influence by systemic components is even higher.

The presence of indirect effects offers a new perspective on the so-called credit spread puzzle. While the systemic risk charge plays a rather substantial role for smaller credit spreads, the relative impact on CDS spreads decreases with the absolute level. Consequently, the answer to the credit spread puzzle might lie in the non-diversifiable risk stemming from the financial system.

Our additional findings correspond to other analyses. CDS spreads not only depend on a firm’s leverage and equity volatility but also on its other characteristics and the market conditions. We find a rebate for the size of a financial institution which may be due to diversification effects, economies of scale or the fact that certain firms are simply too big to fail. Furthermore, market liquidity measured by the bid-ask spread plays an important role. Changes in spreads are largely driven by the abnormal return of companies’ stock and, again, market liquidity.

The magnitude of risk propagation not only stresses the role of contagion as a determinant of CDS premiums but also highlights the need for macroprudential supervision, since the failure of a single entity may threaten the whole system. Historically, financial regulation has concentrated on ensuring the stability of each individual financial institution and neglected the risk stemming from the financial system as a whole. Recent advances in the regulatory framework (Basel III, Solvency II) already address the issue of countercyclical buffers counteracting the overdrawn market movements arising from both negative and positive financial spill overs. Our analysis shows that systemic risk is an important risk factor in today’s economic and financial system, arguing for a further step towards macroprudential regulation. The findings also serve as a warning to national and regional authorities which have not yet geared their regulation towards a macro perspective. In this kind of a regulatory framework, financial institutions may possibly over-invest, which is then particularly painful in the event of a crisis.
Conclusion

Of course, this study focuses on the threats of connectivity patterns between financial institutions, yet neglects the potential implications of the concentration of risks stemming from massively writing CDS. Though a lively debate in the literature, this subject falls outside of the scope of this analysis [Stulz 2010].
Appendix A

List of observed financial institutions

<table>
<thead>
<tr>
<th>Name</th>
<th>Abbreviation</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>AEGON</td>
<td>AEGON N.V.</td>
<td>Insurer</td>
</tr>
<tr>
<td>ALZ</td>
<td>Allianz SE</td>
<td>Insurer</td>
</tr>
<tr>
<td>AVLN</td>
<td>AVIVA Plc.</td>
<td>Insurer</td>
</tr>
<tr>
<td>BBVASM</td>
<td>Banco Bilbao Vizcaya Argentaria S.A.</td>
<td>Bank</td>
</tr>
<tr>
<td>CINC</td>
<td>Citi Inc.</td>
<td>Bank</td>
</tr>
<tr>
<td>CRDIT</td>
<td>Unicredit S.A.</td>
<td>Bank</td>
</tr>
<tr>
<td>CRDSUI</td>
<td>Credit Suisse AG</td>
<td>Bank</td>
</tr>
<tr>
<td>DB</td>
<td>Deutsche Bank AG</td>
<td>Bank</td>
</tr>
<tr>
<td>ING</td>
<td>ING Groep N.V.</td>
<td>Bank</td>
</tr>
<tr>
<td>JPMCC</td>
<td>JP Morgan Chase Corp.</td>
<td>Bank</td>
</tr>
<tr>
<td>MIZUHOB</td>
<td>Mizuho Financial Group</td>
<td>Bank</td>
</tr>
<tr>
<td>MS</td>
<td>Morgan Stanley</td>
<td>Bank</td>
</tr>
<tr>
<td>MUFG</td>
<td>Mitsubishi UFJ Financial Group</td>
<td>Bank</td>
</tr>
<tr>
<td>SANTANDER</td>
<td>Banco Santander S.A.</td>
<td>Bank</td>
</tr>
<tr>
<td>SUMIBK</td>
<td>Sumitomo Mitsui Financial Group</td>
<td>Bank</td>
</tr>
</tbody>
</table>

Appendix B

Discussion of potential misspecification of the SAR-Model

In this section, we elaborate on potential misspecifications of the SAR-Model since the appropriateness of the estimation results depends critically on the structure of the residuals. Indeed, the SAR-Model assumes that the residuals are identical and independent distributed. We, however, have not yet verified the appropriateness of the assumption.

In our setting - i.e. spatial panel with fixed individual effects - three dependence structures within the residuals seem possible,

1. spatial autocorrelation,

2. time-wise correlation and

3. heteroskedasticity.
Spatial autocorrelation in the residuals

In order to address the question of spatially autocorrelated residuals, we estimate a SAC-Model. In comparison to the SAR-Model, the SAC-Model takes spatially correlated residuals into account.

\[ \begin{align*}
    y &= \rho Wy + X\beta + u \\
    u &= \lambda Wu + \varepsilon \\
    \varepsilon &\sim N(0, \sigma^2 I_N)
\end{align*} \]

The SAC-Model nests the SAR-Model, thus testing for the significance of \( \lambda \) is a formal straightforward way to test for the presence of spatial autocorrelation in the residuals.

In Table 3.6, we show the estimation results and compare the SAR-Model with the SAC-Model. Indeed, we find evidence for spatially autocorrelated residuals since the hypothesis of \( H_0 : \lambda = 0 \) cannot be rejected. This points towards model misspecification. At the same time, the estimation results of the sar2 and sac2 model are rather identical, leading to the conclusion that type of misspecification is not that relevant for obtaining adequate results. However, a more detailed analysis shows that some variables are biased in the SAR specification. This holds especially true for the yield level and the bid-ask spread. Moreover, the spatial autocorrelation parameter \( \rho \) jumps from 0.13 to 0.27 highlighting ones again the importance of spatial correlation in our model.

Spatial and time-wise autocorrelation and heteroskedasticity

When analysing the SAC-Model in more detail one finds time dependent and heteroskedastic residuals. This can be seen graphically in Figure 3.4 where we plot the standard deviation of the residuals within a month (\( \sigma_t \)). The standard deviation is obtained by calculating the standard deviation of the residuals of the 15 individuals within one fixed point in time (\( t \)),

\[ \sigma_t^2 = \frac{1}{n-1} \sum_{i=1}^{n=15} (e_{i,t} - \hat{e}_t)^2. \]

The time-series plot highlights two major weaknesses of the SAC-Model. Firstly, the standard deviation of the residuals is time dependent. For example, in the crisis period the average standard deviation jumps from 15 to 100 basis points. A structural break of the volatility of the residuals in mid 2007 is evident. Secondly, in the period from

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CDS spreads and systemic risk

2004 to 2007 the average standard deviation shows a periodic behaviour pointing towards autocorrelated residuals.

Figure 3.4: Plot of the median standard deviation of the residuals. Ribbons indicate the .75 and .25 percentiles.

Even though the SAC-Model takes spatial autocorrelation in the residuals into account, we do not find identical and independent distributed residuals. Thus we have to come up with a model which is able to handle heteroskedastic innovations,

\[
\begin{align*}
    y &= \rho Wy + X\beta + u \\
    u &= \lambda Wu + \varepsilon \\
    \varepsilon_{i,t} &\sim N(0, \sigma_{i,t}^2).
\end{align*}
\]

This model allows for both a spatial lag in the dependent as well as in the disturbances. Moreover, the innovations in the disturbance process are assumed heteroskedastic of an unknown form. In econometrics are two different approaches applied to address heteroskedastic innovations of an unknown form. The first method applies a heteroskedasticity correction to the coefficients covariances. This method does not touch the parameter estimates since these are unbiased. However, the covariances are corrected. The estimation results of this approach can be seen in Table 3.6 and is labelled with robust s.e. where s.e. stands for standard errors. The second solution to the problem is to implement a non-parametric heteroscedasticity and autocorrelation consistent (HAC) estimator. In a spatial context, this approach was proposed by Kelejian and Prucha (2007).
The comparison of the results of the sac2 model, the sac2 model with robust s.e. and the sac2-HAC model, leads to the conclusion that the estimation results are rather robust since the regression coefficients and the standard errors are insensitive with respect to the different estimation procedures.

<table>
<thead>
<tr>
<th>Variable</th>
<th>ls2</th>
<th>sar2</th>
<th>sac2</th>
<th>sac2</th>
<th>sac2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(robust s.e.)</td>
<td>(HAC)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.21)</td>
<td>(3.19)</td>
<td>(3.54)</td>
<td>(4.17)</td>
<td>(-4.22)</td>
</tr>
<tr>
<td>Slope</td>
<td>5.91</td>
<td>5.33</td>
<td>5.72</td>
<td>5.72</td>
<td>5.75</td>
</tr>
<tr>
<td></td>
<td>(3.20)</td>
<td>(3.17)</td>
<td>(3.44)</td>
<td>(3.49)</td>
<td>(3.49)</td>
</tr>
<tr>
<td>Curvature</td>
<td>14.01***</td>
<td>13.99***</td>
<td>14.60***</td>
<td>14.60**</td>
<td>14.61***</td>
</tr>
<tr>
<td></td>
<td>(3.49)</td>
<td>(3.43)</td>
<td>(3.69)</td>
<td>(4.92)</td>
<td>(4.95)</td>
</tr>
<tr>
<td>$\sigma_{\text{Equity}}$</td>
<td>4.48***</td>
<td>4.55***</td>
<td>5.04***</td>
<td>5.04***</td>
<td>5.05***</td>
</tr>
<tr>
<td></td>
<td>(0.76)</td>
<td>(0.75)</td>
<td>(0.78)</td>
<td>(1.29)</td>
<td>(1.29)</td>
</tr>
<tr>
<td>log(assets)</td>
<td>-33.78***</td>
<td>-35.20***</td>
<td>-39.11***</td>
<td>-39.11***</td>
<td>-39.17***</td>
</tr>
<tr>
<td></td>
<td>(5.96)</td>
<td>(5.87)</td>
<td>(6.07)</td>
<td>(6.89)</td>
<td>(6.90)</td>
</tr>
<tr>
<td>Leverage</td>
<td>6.63***</td>
<td>6.65***</td>
<td>6.53***</td>
<td>6.53***</td>
<td>6.53***</td>
</tr>
<tr>
<td></td>
<td>(0.87)</td>
<td>(0.86)</td>
<td>(0.89)</td>
<td>(1.68)</td>
<td>(1.68)</td>
</tr>
<tr>
<td>Libor-OIS spread</td>
<td>52.85***</td>
<td>51.40***</td>
<td>56.41***</td>
<td>56.41*</td>
<td>56.67**</td>
</tr>
<tr>
<td></td>
<td>(5.62)</td>
<td>(5.73)</td>
<td>(6.10)</td>
<td>(22.02)</td>
<td>(22.19)</td>
</tr>
<tr>
<td>Bid-ask spread</td>
<td>6.14***</td>
<td>4.42***</td>
<td>1.65</td>
<td>1.65</td>
<td>1.63</td>
</tr>
<tr>
<td></td>
<td>(0.67)</td>
<td>(0.82)</td>
<td>(1.12)</td>
<td>(1.56)</td>
<td>(1.56)</td>
</tr>
<tr>
<td>Rating A–</td>
<td>24.91***</td>
<td>22.58***</td>
<td>18.35**</td>
<td>18.35***</td>
<td>18.31***</td>
</tr>
<tr>
<td></td>
<td>(6.35)</td>
<td>(6.27)</td>
<td>(6.41)</td>
<td>(4.97)</td>
<td>(4.95)</td>
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<tr>
<td></td>
<td>(5.14)</td>
<td>(5.14)</td>
<td>(5.31)</td>
<td>(4.92)</td>
<td>(4.82)</td>
</tr>
<tr>
<td>Rating AA</td>
<td>-58.85***</td>
<td>-56.78***</td>
<td>-54.61***</td>
<td>-54.61***</td>
<td>-54.62***</td>
</tr>
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<td></td>
<td>(8.05)</td>
<td>(7.95)</td>
<td>(8.00)</td>
<td>(9.97)</td>
<td>(9.98)</td>
</tr>
<tr>
<td>Rating AA–</td>
<td>-50.08***</td>
<td>-46.76***</td>
<td>-43.64***</td>
<td>-43.64***</td>
<td>-43.68***</td>
</tr>
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<td></td>
<td>(6.23)</td>
<td>(6.24)</td>
<td>(6.40)</td>
<td>(8.40)</td>
<td>(8.51)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.13**</td>
<td>0.27***</td>
<td>0.27*</td>
<td>0.27*</td>
<td>0.27*</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.06)</td>
<td>(0.11)</td>
<td>(0.11)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>-</td>
<td>-</td>
<td>0.17**</td>
<td>0.17**</td>
<td>0.27**</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>(0.04)</td>
<td>(0.06)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>N Obs</td>
<td>1080</td>
<td>1080</td>
<td>1080</td>
<td>1080</td>
<td>1080</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.75</td>
<td>0.74</td>
<td>0.73</td>
<td>0.73</td>
<td>0.73</td>
</tr>
</tbody>
</table>

Table 3.6: Comparison of SAR and SAC-Model; standard errors in parentheses; * significant at the 5% level; ** significant at the 1% level; *** significant at the 0.1% level.
Appendix C

Sensitivity analysis of the estimation results with respect to changes in $W$

Since the construction of the weighting matrix has been subject to heated debates in the past, we provide a sensitivity analysis of the estimation results with respect to changes within $W$. For this purpose we shock each element $w_{i,j}$ $\forall i \neq j$ of the matrix $W$ by a uniformly distributed random number $s \in [0.98, 1.02]$. Due to the symmetry of the weighting matrix $W$, we apply a symmetric shock, i.e. $s_{i,j} = s_{j,i}$.

$$s_{i,j} \sim U(0.98, 1.02)$$

In order to quantify the sensitivity of the estimation results, we execute a simulation. In each simulation, the specific element $w_{i,j}$ of $W$ is multiplied with its random factor $s_{i,j}$. Subsequently, we re-estimated the coefficients of the sar2 model. This procedure is repeated 1000 times. Finally, we calculated the mean, minimum and maximum of the simulated coefficients.

In Table 3.7, the results of the sensitivity analysis are reported. The first three columns of Table 3.7 show the minimum, mean and maximum of each coefficient of the sar2 model respectively. Columns four and five show the minimum/mean and maximum/mean ratio. From Table 3.7 one can infer that the reported results are rather robust with respect to variations within the weighting matrix $W$ since the impact on the regression coefficients is about one-tenth of the original shock.
<table>
<thead>
<tr>
<th>Variable</th>
<th>min</th>
<th>mean</th>
<th>max</th>
<th>$c_{\text{min}}/c_{\text{mean}}$</th>
<th>$c_{\text{max}}/c_{\text{mean}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.125</td>
<td>0.127</td>
<td>0.131</td>
<td>0.980</td>
<td>1.025</td>
</tr>
<tr>
<td>Level</td>
<td>-5.429</td>
<td>-5.420</td>
<td>-5.411</td>
<td>1.002</td>
<td>0.998</td>
</tr>
<tr>
<td>Slope</td>
<td>5.311</td>
<td>5.328</td>
<td>5.344</td>
<td>0.997</td>
<td>1.003</td>
</tr>
<tr>
<td>Curvature</td>
<td>13.975</td>
<td>13.988</td>
<td>13.997</td>
<td>0.999</td>
<td>1.001</td>
</tr>
<tr>
<td>$\sigma_{\text{Equity}}$</td>
<td>4.544</td>
<td>4.549</td>
<td>4.555</td>
<td>0.999</td>
<td>1.001</td>
</tr>
<tr>
<td>log(assets)</td>
<td>-35.242</td>
<td>-35.196</td>
<td>-35.163</td>
<td>1.001</td>
<td>0.999</td>
</tr>
<tr>
<td>Leverage</td>
<td>6.647</td>
<td>6.649</td>
<td>6.652</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Libor-OIS spread</td>
<td>51.337</td>
<td>51.401</td>
<td>51.451</td>
<td>0.999</td>
<td>1.001</td>
</tr>
<tr>
<td>Bid-ask spread</td>
<td>4.382</td>
<td>4.422</td>
<td>4.456</td>
<td>0.991</td>
<td>1.008</td>
</tr>
<tr>
<td>Rating A−</td>
<td>22.517</td>
<td>22.581</td>
<td>22.640</td>
<td>0.997</td>
<td>1.003</td>
</tr>
<tr>
<td>Rating A+</td>
<td>-36.203</td>
<td>-36.130</td>
<td>-36.052</td>
<td>1.002</td>
<td>0.998</td>
</tr>
<tr>
<td>Rating AA</td>
<td>-56.854</td>
<td>-56.781</td>
<td>-56.728</td>
<td>1.001</td>
<td>0.999</td>
</tr>
<tr>
<td>Rating AA−</td>
<td>-46.840</td>
<td>-46.759</td>
<td>-46.681</td>
<td>1.002</td>
<td>0.998</td>
</tr>
</tbody>
</table>

Table 3.7: Sensitivity analysis of the estimation results of the sar2 model with respect to variations in $W$. 
4 Interest rate risk and the Swiss Solvency Test

Authored by: Armin Eder, Sebastian Keiler, Hannes Pichl

4.1 Introduction

As 2011 began, and after a five-year transition period, the Swiss Solvency Test (SST) came into force requiring insurance undertakings to meet regulatory capital requirements. The quantitative aspects of this new regulatory framework are based on market-consistent valuation of assets and liabilities and on solvency capital requirements which reflect the risk profile of the undertakings’ balance sheets and the underwritten business.

Solvency is measured by comparing the required amount of solvency capital with the amount of available capital. Within the framework of the SST, available capital is referred to as Risk Bearing Capital ($RBC$). The $RBC$ can be interpreted as the undertaking’s capacity to write new business and to absorb future losses. At a given point in time, the $RBC$ is calculated on the basis of the balance sheet of a company. The $RBC$ is derived as the sum of the net asset value ($NAV$) of the market-consistent balance sheet and – if existing – of subordinated hybrid capital less possible capital deductions such as anticipated future dividends.

For the assessment of an undertaking’s solvency position under the SST, the existing capacity of a company to absorb losses and to cope with risks in general needs to be

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1. This essay is part of Eder, A., Keiler, S., and Pichl, H. (2013). Interest Rate Risk and the Swiss Solvency Test. *Bundesbank Discussion Paper*, (41/2013) and Eder, A., Keiler, S., and Pichl, H. (2014b). Interest rate risk and the swiss solvency test. *European Actuarial Journal*, pages 1–23. This would not have been possible without the helpful comments and remarks we received on several occasions when we have presented this work.

2. Modern insurance regulation typically requires a market-consistent valuation of insurance liabilities which in turn is often achieved by means of stochastic valuation. However, the $RBC$ is given at any point in time once the expected time value of management rules and policyholder behaviour was analysed.
Introduction

confronted with a quantity that captures what could go wrong in the future with the company within a certain period of time. The qualitative question is quantitatively specified by the definition of the required amount of solvency capital, the so called Target Capital ($TC$), as the Expected Shortfall ($ES$) of the undertaking’s aggregate loss distribution at the one-percent quantile after a period of one year ($ES_{0.01}$). An insurance undertaking meets solvency if the RBC exceeds the $TC$, i.e. if $RBC \geq TC$. Without too much formal rigour this definition can be understood as follows: the SST requires entities to hold enough available capital that, out of one hundred companies with a solvency coverage ratio $\geq 100\%$ at $t = 0$, the average number of companies defaulting within one year is less than one.

The determination of the aggregate loss distribution of a given undertaking after one year is at the centre of the calculation of the $TC$. While the $RBC$ is observable at any given point in time, $RBC_{t=0} := RBC_0$, the value of $RBC$ at $t = 1$ is unknown at $t = 0$. The $RBC$ at $t = 1$ is therefore a random variable. From the distribution of $RBC_{t=1}$, the aggregate loss distribution $\Delta RBC_{t=1} = v \cdot RBC_{t=1} - RBC_0$ of the undertaking can be calculated and the $ES$ at the one-percent quantile can be read.$^3$ The factor $v = 1/(1+r_1)$ discounts the $RBC_{t=1}$ to period $t = 0$ with a risk-free one-year spot rate. In this paper, it is assumed that losses arise because of adverse deviations from expectations about the future. Within the framework of the SST, losses may result from changes in the market-consistent valuation of assets and liabilities, from deviations from the expected result of underwriting insurance business and counterparty defaults. These losses need to be recognised in the aggregate loss distribution. Stated slightly differently, the SST considers a set of risk factors as the sources of possible losses in available capital. These risk factors include financial market risks, technical underwriting risks and counterparty default risks. One of the most prominent sources of risk is the volatility of interest rate levels. Interest rate risk refers to changes in risk-free interest rates. In the SST, risk-free interest rates are derived from government bond yields. The role and the measurement of the associated interest rate risk is the subject of this paper.

Interest rates are of twofold relevance within the SST. Firstly, the importance of interest rates and their term structure derives from the simple fact that there is no market value for insurance liabilities, at least not for all kinds of liability.$^5$ Yet, the SST requires the assignment of a market value to insurance liabilities ($MVL$). This problem is circumvented

---

$^3$ For further details the reader is referred to Swiss Financial Market Supervisory Authority (2006) and Keller and Luder (2004).

$^4$ Strictly speaking, the SST loss distribution constitutes an aggregation of a continuous loss distribution and losses generated by specific scenarios with predefined occurrence probabilities.

$^5$ Liabilities deriving from pure linked business may be valued directly by the market value of the related assets.
Interest rate risk and the Swiss Solvency Test

operationally by approximating the market value of insurance liabilities through the sum of the best estimate of insurance liabilities (BEL) and a market value margin (MVM): 
\[ MVL \approx BEL + MVM. \]
Both the BEL and the MVM depend on the underlying interest rates and their term structure\[6\]. Therefore, the volatility of interest rate levels affects the SST through its impact on the valuation of insurance liabilities. However, this kind of RBC volatility is determined by observable interest rate changes from one point in time to another.

Secondly, and of more relevance for the present paper, risk-free interest rates themselves and the modelling of future risk-free interest rates typically are of great significance for the amount of required capital. It is natural that an undertaking’s exposure to risk-free interest rate volatility depends on that company’s business model and asset-liability policy. For instance, a company writing short-tailed P&C business is obviously much less exposed to risk-free interest rate risk than a company with, say, a moderate quality ALM policy engaged in traditional life business. For the latter, risk-free interest rate risk may easily become the dominant driver of TC.

As stated before, in this paper we are concerned with measuring interest rate risk and calculating of the associated solvency capital. In this context, we explain which properties an economically sensible solvency regulation should have in our opinion and we briefly outline what we consider to be the shortcomings of the SST standard model. This synopsis is followed by a detailed presentation of our approach to model interest rate risk.

4.1.1 The standard model in a nutshell

When the SST was initiated, the standard model for financial market risks started with a loss distribution that was based on a multilinear approximation of the \( RBC_{t=1} \) at \( RBC_{t=0} \) with respect to market risk factors \( x_{1,t}, \ldots, x_{n,t} \). The \( RBC \) at \( t = 1 \) read

\[
RBC(x_{1,t=1}, \ldots, x_{n,t=1}) = RBC(x_{1,t=0} + \Delta x_1, \ldots, x_{n,t=0} + \Delta x_n),
\]

(4.1)

\[6\] It might be argued that the BEL and the MVM are communicating vessels in terms of the impact of the underlying interest rates and that it is only the sum of both which matters (Keller et al., 2011). However, calculating of the MVM is a delicate problem in its own right and practical experience indicates that both quantities are not two sides of the same coin to an extent that would the basis for a theory.

In the SST, the \( RBC \) is calculated using only the BEL rather than the full market value of liabilities MVL. The MVM becomes part of the TC. Instead of reducing the \( RBC \) by the MVM, the SST increases the TC by the MVM, which is conceptually somewhat fuzzy. Contrary to the SST, the new European Solvency II explicitly treats both BEL and MVM as components of the market value of insurance liabilities; hence, the existence of an MVM leads directly to a reduction of available capital rather than to an increase in required capital.
which was approximated by a linear function as follows (with all derivatives calculated at $t = 0$)\footnote{In the currently applicable version of the standard model, the linear expansion of $RBC_{t=1}$ was replaced by a second order Taylor expansion in all market risk factors at $RBC_{t=0}$.
}

$$RBC(x_{1,t=1}, \ldots, x_{n,t=1}) \approx RBC(x_{1,t=0}, \ldots, x_{n,t=0}) + \sum_{i=1}^{n} \frac{\partial RBC}{\partial x_i} \cdot \Delta x_i. \quad (4.2)$$

The marginal distributions of each market risk factor $x_i$ were assumed to be Gaussian distributions. The entire loss distribution for all market risks under consideration was modelled as a multivariate Gaussian distribution which had the advantage that the TC, i.e. the Expected Shortfall on a one per cent level ($ES$), could be calculated analytically with random realisations of all risk factors being drawn as one vector from a multivariate Gaussian distribution as shown in Equation 4.3\footnote{The Expected Shortfall is defined as $ES_\alpha = \frac{1}{\alpha} \int_{0}^{\alpha} VaR_{1-\gamma}(x) d\gamma$, where $VaR_\gamma$ denotes the Value at Risk, i.e. the $\gamma$-quantile.}

$$\Delta x \sim N(0, \Sigma) \quad (4.3)$$

$\Delta x$ indicates the return or the change of the risk factor, respectively, i.e. the first difference $\Delta x = x_{t=1} - x_{0}$\footnote{The SST standard model uses log-differences for non-interest rate risk factors, and first differences of interest rate risk factors. Since we deal with interest rate risk we only consider first differences.}

As an example, the parameterisation of representative interest rate risk factors as used for the SST as of year end 2011 is shown in Table 4.1 below.

<table>
<thead>
<tr>
<th>Time to maturity</th>
<th>Zero rate $x_{t=0}$</th>
<th>Volatility</th>
<th>$ES_{0.01}(x_{t=1})$</th>
<th>Observed minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>15.5</td>
<td>60.3</td>
<td>-145.2</td>
<td>1.2</td>
</tr>
<tr>
<td>5 years</td>
<td>19.9</td>
<td>58.7</td>
<td>-136.5</td>
<td>-5.0</td>
</tr>
<tr>
<td>10 years</td>
<td>74.0</td>
<td>55.2</td>
<td>-73.1</td>
<td>53.0</td>
</tr>
<tr>
<td>30 years</td>
<td>163.9</td>
<td>53.3</td>
<td>21.8</td>
<td>105.6</td>
</tr>
</tbody>
</table>

Table 4.1: Parametrisation of the marginal distribution of four interest rate risk factors along with the expected shortfall estimate at the one percent level. Numbers are quoted in basis points.

The standard model undoubtedly has the advantage of being simple and immediately comprehensible. On the other hand, the example of the expected shortfall in Table 4.1

\footnote{Throughout this paper, bold symbols indicate vectors or matrices.}
demonstrates that the standard model leads to quite disputable results, in particular in a low interest rate environment such as the one prevailing at present. An expected shortfall of roughly \(-150\) basis points for the one year interest rate seems remarkably low and is, from an economic perspective, rather hard to justify.

4.1.2 Shortcomings of the standard model

In general, we are convinced that the standard model is marred by various shortcomings, in particular with respect to the treatment of risk-free interest rates. These shortcomings can be summarized as follows:

- Allowance for significantly negative interest rates.
- Procyclicality of capital requirement: in the standard model, the relation between the level of risk-free interest rates and the required solvency capital tends to be procyclical. When evaluating the interest rate risk of a given undertaking with positive duration of the RBC in the standard model, one finds that the company’s TC in a low-interest rate environment essentially stays the same as in the case of higher interest rate environment. Often, the liabilities’ duration of insurance undertakings are longer than their assets, therefore the solvency capital requirement is driven by low interest rates. Typically, this is the case for life insurance companies. In our opinion, the invariance of the Target Capital is problematic as in a low-interest rate environment the capital requirement should drop significantly as the down-side potential of already low interest rates is very limited and might ultimately be determined by the cost of holding cash. On the other hand, the down-side potential of high interest rates should be reflected in higher capital requirements. Hence, the standard model leads to a capital requirement for interest rate risk in a low-interest rate environment that is too high relative to the capital requirement within a high-interest rate environment. This behaviour is attributable to the use of Gaussian distributions for modelling interest rate risk, which at the same time leads to a robust, but economically unjustified capital requirement. In short, we believe that companies should be required to hold more capital in good times and less in bad times.

\[\text{Rather than using the standard Macauley definition with a negative sign, we define the duration of the RBC as the partial derivative of the RBC with respect to the interest rate. Consequently, a positive duration of the RBC implies losses in RBC in case of decreasing interest rates.}\]

\[\text{In fact, it is true that the TC would actually increase because of the convexity of the RBC, i.e. due to increasing duration in line with decreasing interest rates.}\]
Methodology towards a new interest rate risk model

- Multitude of risk factors: the standard model incorporates significantly more interest rate risk factors than are actually needed. This is due to the fact that the functional dependence between the term structure of interest rates and the underlying models used to produce these term structures is not reflected in the standard model at all. When combined with stochastic valuation techniques for life business (similar to market-consistent embedded value calculations), the presence of more interest rate risk factors than necessary leads to costly and time consuming numerical simulations which are not essential for modelling interest rate risk.

In this paper, we address the aforementioned shortcomings of the SST standard model and present ways to improve measuring interest rate risk. Under the current regulatory framework of the SST, these improvements would have to be implemented within a company-specific internal risk model.

4.2 Methodology towards a new interest rate risk model

The development of a generic risk model involves three important steps. The first step consists in identifying the risk drivers. The second step is setting up a model that describes the evolution of these risk drivers over time. And the third step is the development of a model parameter estimator. This section concentrates on the realisation of this programme with respect to interest rate risk. First, we will isolate the interest rate risk factors. After that, we will introduce the risk model and the parameter estimation.

4.2.1 Interest rate risk factors

In order to introduce interest rate risk within a risk model, the term structure of interest rates – the functional relationship between time to maturity and the spot rate – needs to be specified. The SST standard model uses thirteen spot rate maturity buckets to model the change of the shape of the yield curve (for each currency in which the undertaking has a material investment). Setting up a risk model in line with the standard model entails the extensive task of modelling all thirteen buckets separately. However, we argue that modelling the spot rates directly is inefficient and may lead to badly behaving yield curves and ignores the functional dependencies across the buckets. An alternative involves reconsidering the data generating process of spot rate curves directly.

\[\text{For comparison see} \] Swiss Financial Market Supervisory Authority (2012).
4.2.1.1 The data generating process of spot rate curves

Typically, we do not observe long-term zero coupon yields directly, nor do we have enough market data on coupon-paying bonds to bootstrap the yield curve. Thus, a term structure model is needed to extract the spot rate curve from the available market data. For example, the Swiss National Bank’s (SNB) spot rates are calculated using the Svensson [1994] model which is calibrated on the basis of market prices and cash-flow patterns of coupon paying bonds [Müller 2002]. FINMA uses exactly these interest rates for valuation purposes within the SST.

The Svensson model is a parsimonious four factor interest rate model and is widely used by central banks such as the ECB or the Deutsche Bundesbank. In the standard model of the SST, SNB spot rates are used for the purpose of valuation and to model Swiss Francs risk-free interest rate risk. The Svensson model assumes that the instantaneous forward rate at time \( t \) is of the following functional form:

\[
f(\tau, \theta_t) = c_{1,t} + c_{2,t} \cdot (e^{-\tau \lambda_{1,t}}) + c_{3,t} \cdot (\tau \lambda_{1,t}e^{-\tau \lambda_{1,t}}) + c_{4,t} \cdot (\tau \lambda_{2,t}e^{-\tau \lambda_{2,t}}).
\]  

(4.4)

In this expression, the parameter vector \( \theta_t \) reads \( \theta_t = [c_{1,t}, c_{2,t}, c_{3,t}, c_{4,t}, \lambda_{1,t}, \lambda_{2,t}] \) and the time to maturity is given by \( \tau \). The resulting functional form of the spot rate curve is then given by:

\[
r(\tau, \theta_t) = c_{1,t} + c_{2,t} \cdot \left( \frac{1 - e^{-\lambda_{1,t} \tau}}{\tau \lambda_{1,t}} \right) + c_{3,t} \cdot \left( \frac{1 - e^{-\lambda_{1,t} \tau}}{\tau \lambda_{1,t}} - e^{-\lambda_{1,t} \tau} \right) + c_{4,t} \cdot \left( \frac{1 - e^{-\lambda_{2,t} \tau}}{\tau \lambda_{2,t}} - e^{-\lambda_{2,t} \tau} \right).
\]  

(4.5)

The spot rate yield curve is fully specified by the mathematics in Equation 4.5. From this it is evident that the vector \( \theta_t \) represents a natural set of risk factors and is better suited for the measurement of interest rate risk than any set of interest rate levels for individual maturities or maturity buckets. The main reason for using \( \theta_t \) as risk factors for the risk model is the fact that the entire space of possible future interest rates and yield curves is included in Equation 4.5. Obviously, it is neither necessary nor even appropriate to model thirteen spot rate buckets if all future interest rates can be produced by no more than

\[14\] The instantaneous forward rate \( f(\tau) \) is obtained from convergence of the maturity of the contract to zero, i.e. \( \lim_{\tau \to 0} f(\tau, \hat{\tau}) = f(\tau) \).

\[15\] Note that \( r(\tau) = \frac{1}{\tau} \int_0^\tau f(s)ds \).
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six parameters for a specific time \( t \). There would be a limitation of this conclusion if the spot rates were derived from a very different term structure model than the Svensson model. However, many empirical examinations of the Svensson model have shown that it is flexible enough to approximate any shape associated with yield data (De Pooter, 2007). This suggests that the interest rate risk factor space per currency of the SST standard model – \( \mathbb{R}^{13} \) – is oversized and can be reduced to \( \mathbb{R}^{6} \) per currency without any loss of generality.

4.2.1.2 Constraining the Svensson model

A closer inspection of Equation 4.5 highlights a potential pitfall. Firstly, no arbitrage arguments suggest that the Svensson model does not admit time varying exponents; hence \( \lambda_1 \) and \( \lambda_2 \) should be perceived as constant model parameters (Filipovic, 2009). Secondly, if \( \lambda_1 \) and \( \lambda_2 \) are rather similar, the individual values of the parameters \( c_{3,t} \) and \( c_{4,t} \) cannot be identified – only the sum of \( c_{3,t} \) and \( c_{4,t} \) is empirically accessible. Additionally, multicollinearity problems arise when the decay parameters \( \lambda_1 \) and \( \lambda_2 \) take extreme values. For example, when the decay parameters approach zero, the factors multiplying \( c_{1,t} \) and \( c_{2,t} \) – the so-called factor loadings – will be highly collinear. As a consequence, the parameters may be estimated to have large values with offsetting signs as reported by Gimeno and Nave (2006). We circumvent these problem by interpreting \( \lambda_{1,t} = \lambda_1 \) and \( \lambda_{2,t} = \lambda_2 \) as constants over time. Consequently, the values for \( \lambda_1 \) and \( \lambda_2 \) have been estimated once for the entire sample period. Filipovic (2009) proposes the parametrisation \( 2 \cdot \lambda_1 = \lambda_2 \) in order to meet no-arbitrage conditions. Our estimation results, however, suggest a ratio of 3 for Switzerland and 2.6 for Germany. Nevertheless, this deviation has been observed for other samples as well (Filipovic 2009).

In this setting the Svensson model is fully linear in its factors. Thus Equation 4.5 can be expressed in terms of factor loadings \( l_i(\tau) \) or by the model matrix \( \mathbf{L}(\tau) = [l_1(\tau), l_2(\tau), l_3(\tau), l_4(\tau)] \). By abuse of notation, we continue to denote the (reduced) parameter vector using the same symbol \( \theta_t = [c_{1,t}, c_{2,t}, c_{3,t}, c_{4,t}] \) and write

\[
\mathbf{r}(\tau, \theta_t) = \mathbf{L}(\tau) \cdot \theta_t + \nu_{t,\tau}, 
\]

16 Many other methods to reduce the dimension of the risk factor space have been proposed in the past. One interesting study addressing the oversized interest rate risk factor space within the SST was done by Ambrus et al. (2011).

17 When the decay parameter approaches zero, the factor loading on \( c_{2,t} \) is \( \lim_{\lambda \to 0} \frac{1-e^{-\tau \lambda}}{\tau \lambda} = 1 \). Thus the parameters \( c_{1,t} \) and \( c_{2,t} \) cannot be reliably identified. See for example De Pooter (2007).

18 We estimated \( \lambda_{CH}^1 = 0.0195 \) and \( \lambda_{CH}^2 = 0.0586 \) for Switzerland and \( \lambda_{DE}^1 = 0.0102 \) and \( \lambda_{DE}^2 = 0.0271 \) for Germany.
where
\[
L(\tau) = \left[ 1, \left( \frac{1 - e^{-\tau \lambda_1}}{\tau \lambda_1} \right), \left( \frac{1 - e^{-\tau \lambda_1}}{\tau \lambda_1} - e^{-\tau \lambda_1} \right), \left( \frac{1 - e^{-\tau \lambda_2}}{\tau \lambda_2} - e^{-\tau \lambda_2} \right) \right].
\] (4.7)

The assumption of constant exponents certainly decreases the model flexibility, but the in-sample analysis in section 4.4.1 demonstrates that the model fit of the constrained Svensson model is adequate for risk measurement purposes. Unlike Equation 4.5, Equation 4.6 contains an error term \( \nu_{t,\tau} \) that depends on the point in time and the time to maturity. The reason for this new error term is obvious. The Svensson model is the original data generating process of the spot rate curve, hence the model produces a perfect fit. The constrained Svensson model, on the other hand, is based on fewer factors than the Svensson model and therefore cannot reproduce the spot rates exactly. For a detailed examination of the Svensson model and the economic interpretation of its factors the interested reader is referred to De Pooter (2007) amongst others.

### 4.2.2 Expansion of the RBC

Having identified the four factors of the Svensson model as the most natural set of risk factors, \( \theta_t = [c_{1,t}, c_{2,t}, c_{3,t}, c_{4,t}] \), we now have to provide a valuation function for \( RBC(\theta_t) \) to determine its variation over time, i.e. to determine the distribution of future gains and losses as a function of the risk factors. Following the SST standard model in this respect, we propose to use a second order Taylor expansion in \( \theta_t \) as the valuation function.

The suggested Taylor expansion of \( RBC_{t=0} \) may be applied in two ways. Firstly, it can be used to calculate the solvency capital requirement for interest rate risk by translating simulated Svensson factors at \( t = 1 \) into the Expected Shortfall of gains and losses of an undertaking due to shifting interest rates. Secondly, the Taylor expansion of \( RBC_{t=0} \) can be used to estimate the new value of \( RBC \) at an instant \( \Delta t \) later, \( RBC_{t=\Delta t} \), by taking into account the factual changes in spot rates that have occurred in the market over a period \( \Delta t \) since the last calculation of the Risk Bearing Capital. For both applications, the link between observed spot rates and the Svensson factors is a prerequisite.\(^{19}\)

When we consider the case of risk-free interest rates for one currency, the – at any point in time – measurable quantity \( RBC_t \) can be expressed by two sets of risk factors. The

\(^{19}\) Remember that the constrained Svensson model is a simplification of the Svensson spot rate model that was the starting point for our analysis. Therefore, the link between spot rate sensitivities and sensitivities with respect to the four Svensson factors can only be an approximate, but highly accurate.
first set of risk factors refers to thirteen spot rates or spot rate buckets as risk factors, \( r_t = [r_{1,t}, \ldots, r_{13,t}] \). In our approach, the same quantity \( RBC \) is expressed by function \( RBC_t \) using the four factors \( c_{i,t} \):

\[
RBC_t = RBC_t(r_{1,t}, \ldots, r_{13,t}) = RBC_t(L(1) \cdot \theta_t, \ldots, L(13) \cdot \theta_t) = RBC_t(\theta_t). \tag{4.8}
\]

The partial derivatives with respect to the two sets of risk factors can easily be transformed into each other; the sensitivity of the Risk Bearing Capital with respect to any of the Svensson factors \( \theta_t \) is a weighted sum of the RBC sensitivities with respect to spot rates, where the respective weights are the factor loadings of the Svensson model:

\[
\frac{\partial RBC_t(\theta_t)}{\partial c_k} = \sum_{\tau} \frac{\partial RBC_t(r_t)}{\partial r(\tau, \theta_t)} \cdot \frac{\partial r(\tau, \theta_t)}{\partial c_k} = \sum_{\tau} \frac{\partial RBC_t(r_t)}{\partial r(\tau, \theta_t)} \cdot l_k(\tau). \tag{4.9}
\]

Another approach to calculate the partial derivative of the \( RBC \) with respect to \( \theta_t \) would be scenario analysis. This approach is especially promising for life insurance undertakings which use computationally burdensome and costly stochastic valuation techniques and are obliged to assess interest rate risk. Consequently, calculating only a few interest rate scenarios instead of dozens of sensitivity runs is advisable. For example, the effect of a shift of the entire yield curve by \( \pm 100 \) basis points, i.e. \( l_1 \cdot (\pm 100) \) basis points, is calculated and the partial derivative of the \( RBC \) with respect to \( c_1 \) is subsequently calculated numerically.

### 4.2.3 The risk model

In the previous subsections we isolated the relevant risk factors and discussed one possible approximation of the valuation function of the \( RBC \). The final component of a risk model is the stochastic process of the risk factors. The SST standard model assumes that the risk factors follow a random walk,

\[
\theta_{t+1} = \theta_t + \epsilon_t. \tag{4.10}
\]

Typically, \( \epsilon_t \) is modelled by a multivariate Gaussian distribution with parameters \( \mu \) and \( \Sigma \). This has two important implications. Firstly, if \( \mu \) and \( \Sigma \) are assumed to be constants, i.e. without any sampling errors, it automatically follows that \( \theta_{t+1} \) is Gaussian as well.\(^{21}\)

---

\(^{20}\) This holds true if no cross derivatives of the \( RBC \) with respect to spot rates exist.

\(^{21}\) Note that the \( \mu \) and \( \Sigma \) are estimated from the data and are subject to sampling errors. As a result, \( \theta_{t+1} \) is t-distributed. Since the t-distribution converges very fast to the Gaussian, it is reasonable to think of \( \theta_{t+1} \) as Gaussian as well. In practice the sampling error is often disregarded which may
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Secondly, since the spot rates are weighted averages of the term structure coefficients $\theta_{t+1}$, they belong to the same class of distributions.

However, the assumption of the error terms’ normality may be inappropriate. From a theoretical perspective, the lower bound of interest rates might be determined by the cost of holding cash. Consequently, a symmetric distribution of interest rates may be misleading, especially in low interest environments. In fact, highly negative spot rates – such as implied by a Gaussian – have never been observed. Yet it is also true that for medium and long-term interest rates the hypothesis of a Gaussian distribution cannot always be rejected (see for example Table 4.2). We address this issue by assuming that the error vector $\epsilon_t$ follows a truncated Gaussian distribution:

$$\epsilon_t \sim \text{TruncNormal}(\mu, \Sigma | \Omega_t)$$

(4.11)

$\mu$ and $\Sigma$ are the location vector and the scale matrix of the truncated multivariate Gaussian distribution. The truncation of the Gaussian is implemented by conditioning on the information set given at time $t$, $\Omega_t$. Qualitatively speaking, the truncation of the error terms’ Gaussian is introduced by taking into account the existence of economically reasonable lower bounds on interest rates. Once any lower bound on interest rates is defined at $t = 0$, it defines the accessible area of the $\epsilon$-space $\Omega_t$ for any given time $t$.

In this paper, the truncation is introduced via conditioning on the instantaneous forward rates defined in Equation 4.4: the instantaneous forwards rates $f(\tau, \theta_{t+1})$ need to be at least equal to $\hat{f}(\tau)$, the original lower bound of the instantaneous forward rates, at any point in time in the future. $\hat{f}(\tau)$ has to be chosen by the company or defined by the regulatory authority:

$$f(\tau, \theta_{t+1}) \geq \hat{f}(\tau).$$

(4.12)

In this setting bold symbol $\tau$ denotes a vector of size $m$ containing relevant interest rate maturities to which the company specific portfolio is sensitive, e.g. $\tau = [0, 12, 24, ..., 600]$ months. We translate the condition of Equation 4.12 into a definition of the accessible area of the $\epsilon$-space, $\Omega_t$, by referring to the random walk of the risk factors $\theta_{t+1}$:

$$A \cdot (\theta_t + \epsilon_t) \geq \hat{f}(\tau).$$

(4.13)

result in too narrow distributions of the predictions. However, the effect may be minor compared with other sources of error (Chatfield, 1993, 2000).

22 If one assumes that $\epsilon_t$ is multivariate Gaussian, one simply ends up with the SST standard model, but with a reduced risk factor space.
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In [Equation 4.13] we introduced the matrix $A$ of size $m \times 4$ of the instantaneous forward rate factor loadings,

$$A = [1, (e^{-\tau \lambda_1}), (\tau \lambda_1 \circ e^{-\tau \lambda_1}), (\tau \lambda_2 \circ e^{-\tau \lambda_2})]$$

(4.14)

where $1$ is a vector of size $m \times 1$ and $\tau$ is a vector of size $m \times 1$; hence $e^{-\tau \lambda_1}$, $\tau \lambda_1 \circ e^{-\tau \lambda_1}$ and $\tau \lambda_2 \circ e^{-\tau \lambda_2}$ are vectors of size $m \times 1$. We further formulate the truncation condition as follows:

$$A \cdot \epsilon_t \geq \hat{f}(\tau) - A \cdot \theta_t.$$  

(4.15)

For further convenience, we define $a_t := \hat{f}(\tau) - A \cdot \theta_t$ and write

$$\Omega_t = \{\epsilon_t | A \cdot \epsilon_t \geq a_t\}.$$  

(4.16)

One should be aware that $a_t$ is a random variable. In this setting the scale and location parameters of the truncated normal distribution are time invariant by assumption but the inequality constraints and hence the integration region are random and vary over time. To demonstrate this more clearly consider the following example.

Let us assume that, at time $t$, the interest rate is high. In such a case, the inequality constraint $A \cdot \epsilon_t \geq a_t$ is only weakly binding and of less relevance. Now consider a significant decrease in interest rates from time $t$ to $t+1$, expressed by a change in risk factors $\Delta \theta_{t+1}$. This exogenous shock of interest rates leads to an increase in the lower bound $a_{t+1}$, $a_{t+1} = a_t - A \cdot \Delta \theta_{t+1}$, which, in consequence, makes the condition $A \cdot \epsilon_{t+1} \geq a_{t+1}$ much stronger binding at $t+1$ than it was at $t$. From this example it should be evident that any change in interest rates also changes the domain of $\epsilon_t$.

4.2.3.1 Properties of the plane-truncated Gaussian distribution

Two properties of this class of plane-truncated Gaussian distributions are vital for the construction of a parameter estimator.

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23 This notation (e.g. $\tau \lambda_1 \circ e^{-\tau \lambda_1}$) introduces the element-wise product (Hadamard product).

24 We allow for time-variant inequality constraints, $\Omega_t$.

25 For more details on the moment generating function of the truncated multi-normal distribution see for example Tallis [1965].
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Property 1 The truncated Gaussian density function $TN_{\epsilon_t}(\mu, \Sigma \mid \Omega_t)$ is proportional to the Gaussian density for $A \cdot \epsilon_t \geq a_t$

$$TN_{\epsilon_t}(\mu, \Sigma \mid \Omega_t) = \begin{cases} \frac{1}{\alpha_t} N_{\epsilon_t}(\mu, \Sigma) & A \cdot \epsilon_t \geq a_t \\ 0 & A \cdot \epsilon_t < a_t \end{cases}$$

The normalisation factor $\alpha_t$ is determined as the integral of the density over the region being specified by $\Omega_t$,

$$\alpha_t = \int_{\Omega_t} N_{\epsilon_t}(\mu, \Sigma) d\epsilon_t.$$ \hfill (4.18)

In Property 1 we introduce the following notation: $N_{\epsilon_t}(\cdot, \cdot)$ denotes the multi-dimensional Gaussian density function of the error term (as indicated by the subscript) $\epsilon_t$ with parameters $\mu$ and $\Sigma$. It is particularly important to keep in mind that the location and scale parameters of the truncated Gaussian distribution are not identical with the expected values and the covariance matrix of the truncated Gaussian.

Property 2 The conditional expectation value of a truncated Gaussian distribution is given by

$$E[\epsilon_t \mid \Omega_t] = \frac{1}{\alpha_t} \int_{\Omega_t} \epsilon_t N_{\epsilon_t}(\mu, \Sigma) d\epsilon_t =: \hat{\mu}_t,$$ \hfill (4.19)

and the conditional covariance matrix is determined by

$$Var[\epsilon_t \mid \Omega_t] = \frac{1}{\alpha_t} \int_{\Omega_t} (\epsilon_t - E[\epsilon_t]) N_{\epsilon_t}(\mu, \Sigma)(\epsilon_t - E[\epsilon_t])^T d\epsilon_t =: \hat{\Sigma}_t.$$ \hfill (4.20)

4.2.3.2 Parameter estimator

We already highlighted that, in our setting, $a_t$ is time-dependent. This has two major implications for estimating the time invariant parameters $\mu$ and $\Sigma$. Firstly, each sample observation differs in the conditional expected value, and secondly, the data is heteroskedastic. This complicates the parameter estimation. However, by exploiting Property 2 we are able to set up a moment estimator with the following moment conditions \[^{26}\]

[^26]: $\hat{\sigma}_{i,j,t}$ and $\hat{\mu}_{i,t}$ are the specific elements of the covariance matrix $\hat{\Sigma}_t$ and the mean vector $\hat{\mu}_t$ respectively. Moreover $g_t(\mu, \Sigma)$ is a vector of size 14.
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Under the assumption of a random walk for $\theta_t$ and under the null hypothesis of $\epsilon_t$ being a conditionally truncated Gaussian, it follows that $E[g_t(\mu, \Sigma)|\Omega_t] = 0$. The idea behind a Generalised Method of Moments (GMM) estimator is to replace the orthogonality condition $E[g_t(\mu, \Sigma)|\Omega_t]$ with its sample analogue,

$$m(\mu, \Sigma) = \frac{1}{T} \sum_{t=1}^{T} g_t(\mu, \Sigma),$$

(4.22)

and to choose parameters in such a way that the following quadratic form is minimised:

$$q(\mu, \Sigma) = m(\mu, \Sigma)^T \cdot W \cdot m(\mu, \Sigma).$$

(4.23)

$W$ is a positive definite matrix that does not depend on $\mu$ and $\Sigma$ but may depend on the data. In this application the number of moment conditions matches the number of estimated parameters. Therefore it suffices to find the parameters which solve $m(\mu, \Sigma) = 0$ or minimise the quadratic form $q(\mu, \Sigma) = m(\mu, \Sigma)^T m(\mu, \Sigma)$. As a result, in our case the matrix $W$ defaults to the identity matrix $I$.

The main hurdle in minimizing Equation 4.23 lies in finding values for $[\hat{\mu}_{i,t}, \hat{\sigma}_{i,j,t}]$ at each point in time since the cumulative density of a plane-truncated multivariate Gaussian has no analytical expression and has to be evaluated numerically. The most straightforward method for evaluating the integrals in Property 2 is by means of Monte Carlo integration. We implemented a rejection sampler to calculate $\hat{\mu}_t$ and $\hat{\Sigma}_t$ numerically at each point in time. The rejection sampler consists of the following steps:

1. Given the parameters $\mu$ and $\Sigma$, simulate $N$ vectors from the multidimensional Gaussian.

\[\text{Equation 4.23}\]

\[\text{The moments of the plane-truncated normal distribution were extensively discussed by Tallis [1965]. Algorithms for approximating the cumulative density of the plane-truncated normal distribution were examined by Börsch-Supan and Hajivassiliou [1993].}\]
2. Those simulations that do not fulfill the constraint $A \cdot \epsilon_t \geq a_t$ have to be rejected.

3. From the remaining sample, the expected value and the covariance matrix are calculated. This leads to estimates of $\hat{\mu}_t$ and $\hat{\Sigma}_t$.

4. The procedure is repeated for each point in time.

Given the parameters $\mu$ and $\Sigma$, the rejection sampler delivers the conditional expected values and the scale matrix at each point in time. If both the sample size $T$ and the number of simulations $N$ approach infinity, the combination of the rejection sampler with the moment estimator leads to consistent parameter estimates of $\mu$ and $\Sigma$.

4.2.3.3 Excursus: The truncated Gaussian process and mean reversion

In this section, we elaborate on an important property of a truncated Gaussian process. For the moment, we shall concentrate solely on a univariate time series process where the innovation $\epsilon_t$ is distributed according to a truncated Gaussian. As an example, consider the 10-year to maturity nominal spot rate ($r$) which is empirically bound from below by $a$. We furthermore assume that the spot rate’s data generating process is given by a non-stationary process, $r_{t+1} = r_t + \epsilon_t$. The innovation $\epsilon_t$ follows a conditionally truncated Gaussian, $TN(\mu, \sigma | \Omega_t)$.

Conditioning on a lower bound leads to the existence of the area $\Omega_t$ and guarantees that the spot rate does not drop below $a$ at any point in time. Hence, the innovation $\epsilon_t$ needs to be drawn from $\Omega_t = \{\epsilon_t | \epsilon_t \geq a - r_t\}$. Assume that today’s spot rate is $r_t = r^*$ such that the conditional expectation of $\epsilon_t$ equals zero, $E[\epsilon_t | \Omega_t] = 0$\(^{28}\). Under these assumptions, a negative exogenous interest rate shock decreases $r_{t+1}$; consequently, the inequality constraint $\epsilon_{t+1} + r_{t+1}$ becomes more strongly binding and the expectation of $\epsilon_{t+1}$ becomes positive $E[\epsilon_{t+1} | \Omega_t] > 0$. To phrase it another way, the accessible $\epsilon$-space becomes more and more restricted from the lower bound and is shifted to higher values of $\epsilon_{t+1}$ as the $\epsilon$-domain becomes more and more restricted. Finally, the interest rate is pulled backwards towards $r^*$.

Conversely, consider that today’s interest rate is $r_t = r^*$ and the interest rate undergoes a positive exogenous shock. This will increase $r_{t+1}$ and will relax the constraint $\epsilon_{t+1} \geq a - r_{t+1}$; hence the expectation of $\epsilon_{t+1}$ becomes negative, $E[\epsilon_{t+1} | \Omega_t] < 0$. Again, the spot rate is pulled towards $r^*$ but now this pull is negative.

\(^{28}\) This necessarily implies that $\mu$ is negative; otherwise, no $r^*$ with $E[\epsilon_t | \Omega_t] = 0$ would exist.
In summary, the presented example describes a non-stationary time series process that is mean-reverting and possesses a long-run mean of $r^*$ \(^{29}\) This brings us to the following property.

**Property 3** The random process $r_{t+1} = r_t + \epsilon_t$ with $\epsilon_t \sim \text{TruncNormal}(\mu, \sigma \mid \Omega_t)$ where $\Omega_t = \{\epsilon_t | \epsilon_t \geq a - r_t\}$ and $r_{t=0} \geq a$ is mean-reverting, if and only if, a quantity $r^*$ exists that solves

$$\int_{a-r^*}^{\infty} \epsilon_t N_{\epsilon_t}(\mu, \sigma) d\epsilon_t = 0.$$ \(^{4.24}\)

The required quantity $r^*$ denotes the long-run mean of the process.

Property \(^3\) highlights an important difference between the process in [Equation 4.11] and the SST standard model. The SST standard model assumes that the risk drivers follow a classical random walk with normal innovations. This leads to symmetric and non mean-reverting interest rate distributions. In sharp contrast, the truncated Gaussian process leads to skewed interest rate distributions in a low interest rate environment; depending on the parameter estimates, the truncated Gaussian process will also drift towards a long-run mean.

### 4.2.4 Model recipe

The previous sections outlined a number of theoretical aspects underlying our methodology towards a new SST interest rate risk model. In this section, we briefly summarise the practical target capital sampling recipe on a step-by-step basis.

1. Monthly spot rate data have to be obtained from a data source (e.g. SNB, Deutsche Bundesbank, Bloomberg, etc.).

2. A time series of the relevant risk factors has to be determined. The relevant risk factors are given by the coefficient vector $\theta_t = [c_{1,t}, c_{2,t}, c_{3,t}, c_{4,t}]$ of the constraint Svensson model. The coefficient vector is determined by ordinary least squares (OLS) regression. First the constants $\lambda^{CH}_1 = 0.0195$ ($\lambda^{DE}_1 = 0.0102$) and $\lambda^{CH}_2 = 0.0586$ ($\lambda^{DE}_2 = 0.0271$) have to be set and subsequently spot rates need to be regressed on the factor loadings \(^{30}\) This procedure has to be repeated for every month.

\(^{29}\) For a detailed proof see the Appendix.

\(^{30}\) Alternatively, the coefficient vector and the time invariant constants $\lambda_1$ and $\lambda_2$ can be obtained by non-linear least squares optimisation.
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3. The parameters $\mu$ and $\Sigma$ are estimated by minimising Equation 4.23.

4. The distribution of the coefficient vector in $t+12$ (to produce the realisations of interest rates after one year) has to be simulated.
   a) The simulation is started with $\epsilon_t \sim N(\mu, \Sigma)$ as the proposal.
   b) The risk factors of the model, i.e., the coefficient vector $\theta^t_{t+12} = [\hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{c}_4]_{t+12}$, need to be predicted within the first iteration $i = 1$.
   c) The truncation constraints on the instantaneous forward rates at $\tau = [0, 12, 24, ..., 600]$ need to be tested; if the instantaneous forward rates are at least higher than the threshold $\hat{f}(\tau)$, i.e. $f(\tau, \theta^t_{t+12}) \geq \hat{f}(\tau)$, the simulated coefficient vector will be accepted.
   d) The number of iterations $i$ has to be increased and steps a) to d) need to be repeated $N$ times in order to obtain a distribution of the Svensson model coefficients.

5. Finally, the sensitivities of Risk Bearing Capital with respect to the Svensson coefficients have to be calculated and can be used for approximating the valuation function of the $RBC$ to determine the Target Capital.

4.3 Data

In this study we use end-of-month government spot rates published by the SNB and the Deutsche Bundesbank. Both central banks use the Svensson (1994) model to extract the zero curve from coupon paying bonds. Hence, the original data generating process of the term structure of interest rates is fully specified by Equation 4.5. Our sample comprises spot rates from January 2000 to October 2012 with maturities ranging from one to thirty years.

Table 4.2 presents descriptive statistics of the spot rates. Since the average spot rate increases with time to maturity, the average yield curve is upward sloping. By contrast, the standard deviations (volatilities) decrease with time to maturity. The hypothesis of spot rates being Gaussian is almost always rejected, as indicated by the p-value of the Jarque-Bera test statistic.\(^\text{31}\) Solely medium-term Swiss interest rates might be Gaussian.

\(^{31}\) The Jarque-Bera test hinges on the Null hypothesis that the sample Skewness and the sample excess Kurtosis are jointly zero and thus match a Normal distribution. For large samples, the test statistic is $\chi^2$ distributed. In small samples, p-values may be obtained from Monte Carlo simulations.
This stands in contrast to the findings of De Pooter (2007) who examined US treasury zero coupon bonds and rejected the normal distribution solely for medium and long-term maturities.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Switzerland Mean</th>
<th>Switzerland SD</th>
<th>Switzerland $\rho_1$</th>
<th>Switzerland JB-p</th>
<th>Germany Mean</th>
<th>Germany SD</th>
<th>Germany $\rho_1$</th>
<th>Germany JB-p</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 months</td>
<td>1.356</td>
<td>1.136</td>
<td>0.980</td>
<td>0.000</td>
<td>2.487</td>
<td>1.498</td>
<td>0.978</td>
<td>0.012</td>
</tr>
<tr>
<td>24 months</td>
<td>1.476</td>
<td>1.094</td>
<td>0.975</td>
<td>0.009</td>
<td>2.676</td>
<td>1.425</td>
<td>0.970</td>
<td>0.027</td>
</tr>
<tr>
<td>36 months</td>
<td>1.637</td>
<td>1.055</td>
<td>0.970</td>
<td>0.075</td>
<td>2.875</td>
<td>1.351</td>
<td>0.965</td>
<td>0.041</td>
</tr>
<tr>
<td>48 months</td>
<td>1.808</td>
<td>1.009</td>
<td>0.968</td>
<td>0.203</td>
<td>3.068</td>
<td>1.281</td>
<td>0.962</td>
<td>0.039</td>
</tr>
<tr>
<td>60 months</td>
<td>1.967</td>
<td>0.965</td>
<td>0.966</td>
<td>0.291</td>
<td>3.249</td>
<td>1.214</td>
<td>0.959</td>
<td>0.027</td>
</tr>
<tr>
<td>72 months</td>
<td>2.109</td>
<td>0.928</td>
<td>0.964</td>
<td>0.292</td>
<td>3.414</td>
<td>1.153</td>
<td>0.957</td>
<td>0.017</td>
</tr>
<tr>
<td>84 months</td>
<td>2.231</td>
<td>0.900</td>
<td>0.963</td>
<td>0.239</td>
<td>3.562</td>
<td>1.098</td>
<td>0.955</td>
<td>0.011</td>
</tr>
<tr>
<td>96 months</td>
<td>2.338</td>
<td>0.880</td>
<td>0.961</td>
<td>0.176</td>
<td>3.695</td>
<td>1.050</td>
<td>0.953</td>
<td>0.008</td>
</tr>
<tr>
<td>108 months</td>
<td>2.429</td>
<td>0.867</td>
<td>0.961</td>
<td>0.125</td>
<td>3.812</td>
<td>1.008</td>
<td>0.951</td>
<td>0.007</td>
</tr>
<tr>
<td>120 months</td>
<td>2.509</td>
<td>0.859</td>
<td>0.960</td>
<td>0.091</td>
<td>3.916</td>
<td>0.972</td>
<td>0.950</td>
<td>0.007</td>
</tr>
<tr>
<td>240 months</td>
<td>2.948</td>
<td>0.894</td>
<td>0.962</td>
<td>0.046</td>
<td>4.437</td>
<td>0.879</td>
<td>0.949</td>
<td>0.032</td>
</tr>
<tr>
<td>360 months</td>
<td>3.121</td>
<td>0.956</td>
<td>0.966</td>
<td>0.052</td>
<td>4.524</td>
<td>0.999</td>
<td>0.953</td>
<td>0.033</td>
</tr>
</tbody>
</table>

Table 4.2: Summary statistic for end-of-month spot rates in percentage points. The sample period is January 2000 to October 2012 (N=154). The columns report the mean, the standard deviation, the first order sample autocorrelation, and the p-value of the Jarque-Bera test statistic for normality of changes in spot rates.

The observation of first order sample autocorrelations close to 1 for all maturities and currencies indicates a high persistence of shocks and might point towards a non-stationary time series process. This would imply that the practitioners’ intuitively appealing assumption of stationary interest rates might be misleading. On the other hand, the high sample autocorrelation may also be driven by the downward trend in yield curve levels observed in the past decade; this is depicted in Figure 4.1.

From Figure 4.1, information about the shape of the yield curve can be inferred as well. For example, the Swiss 1-year spot rate exceeded the 5-year spot rate towards the end of 2008 and in 2012; hence the Swiss yield curve was hump-shaped. The German government curve was almost flat at the beginning of 2008. Moreover, the sharp decrease in the interest rate level towards the end of 2008 is impressive. For example, the Swiss 1-year interest rate dropped from 2.408% (October 2008) to 0.691% (December 2008) and has not reverted to its original level since that date. Finally, Swiss interest rate levels are, on average, lower than the German levels.
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Figure 4.1: Time series plots of a subset of maturities of end-of-month zero coupon yields. The sample period is January 2000 to October 2012 (N=154).

4.4 Results

In this section, we empirically examine the proposed framework for modelling interest rate risk in detail. First, we will assess the in-sample fit of the constrained Svensson model. Next, we analyse the model’s capability to replicate observed interest rate distributions. Finally, we focus on simulating the interest rate distributions for $t + 12$ as the basis for deducing the model’s effect on solvency capital requirements. The result is then compared with the SST standard model. For the entire analysis we set the constants in the model matrix $A$ to $\lambda_{1}^{CH} = 0.0195$ and $\lambda_{2}^{CH} = 0.0586$ for Switzerland and $\lambda_{1}^{DE} = 0.0102$ and $\lambda_{2}^{DE} = 0.0271$ for Germany.

For the choice of the instantaneous forward rate threshold $\hat{f}(\tau)$, it is preferable to rely on empirical guidance. However, studies covering this issue seem to be rather scarce and quantifying the cost of holding money is well beyond the scope of this paper. However, it may be reasonable to suggest a lower bound that is governed by interest rates on short-term deposits during low interest rate economic environments – such as the one prevailing today. For Germany, this rate was between 0 and 0.5% in 2013 (Deutsche Bundesbank 2013). However, the actual bound strongly depends on the underlying reference rate. We observe a minimum yield on German government bonds of $-0.1\%$ in our data at one year time to maturity and an in-sample minimum for Swiss government bonds at $-0.38\%$ for the two year to maturity bond. Therefore, it may be reasonable to regard this as the lower bound for the own rate of money. This is in-line with the Swiss Financial Market Supervisory Authority (2012) setting this threshold at $-0.5\%$ for financial market
Results

scenarios. For practical reasons, we opt for this threshold since it corresponds to regulatory guidance.

4.4.1 In-sample fit

A simplification of a spot rate model such as the one set out in Section 4.2.1.2 will of course lead to a deteriorating model fit. In order to assess the implications arising from the application of the parsimonious model we estimate the Svensson model coefficients using ordinary least squares and compare the model predictions with the actual yield curve based on Svensson (1994). Table 4.3 shows the in-sample fit summary statistics of the constrained Svensson model estimated over the time period January 2000 to October 2012. The model has been fitted for each month $t$ and the error is calculated as the difference between the model estimate and the yield curve published by the Deutsche Bundesbank or the Swiss National Bank.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Switzerland</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>q10</td>
<td>q90</td>
<td>SD</td>
</tr>
<tr>
<td>12 to 360 months</td>
<td>-1.86</td>
<td>1.79</td>
</tr>
<tr>
<td>12 months</td>
<td>-1.62</td>
<td>1.16</td>
</tr>
<tr>
<td>24 months</td>
<td>-2.78</td>
<td>3.79</td>
</tr>
<tr>
<td>36 months</td>
<td>-1.42</td>
<td>0.69</td>
</tr>
<tr>
<td>48 months</td>
<td>-1.80</td>
<td>1.12</td>
</tr>
<tr>
<td>60 months</td>
<td>-2.27</td>
<td>2.13</td>
</tr>
<tr>
<td>72 months</td>
<td>-1.96</td>
<td>2.04</td>
</tr>
<tr>
<td>84 months</td>
<td>-0.73</td>
<td>1.23</td>
</tr>
<tr>
<td>96 months</td>
<td>-0.65</td>
<td>0.59</td>
</tr>
<tr>
<td>108 months</td>
<td>-1.19</td>
<td>1.17</td>
</tr>
<tr>
<td>120 months</td>
<td>-2.17</td>
<td>2.13</td>
</tr>
<tr>
<td>240 months</td>
<td>-2.57</td>
<td>1.45</td>
</tr>
<tr>
<td>360 months</td>
<td>-1.88</td>
<td>2.62</td>
</tr>
</tbody>
</table>

Table 4.3: In-sample fit: summary statistics in basis points, estimation window from January 2000 to October 2012. The table reports the 1st and 9th decile of the residuals, as well as the standard deviation and the mean absolute error.

Generally speaking, the summary statistics presented in Table 4.3 show that constraining the Svensson model does not harm the model fit. The first three columns present the in-sample fit of the re-engineered Swiss yield curve. A mean absolute error (MAE) of 1.2 basis points over all maturities indicates rather satisfying properties of the methodology. For Germany, the MAE is somewhat higher and peaks at maturities around two years.
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However, an average absolute error of 1.3 basis points still indicates pleasing nature of the applied approximation. We therefore conclude that for an adequate risk model it suffices to model $\theta_t = [c_{1,t}, c_{2,t}, c_{3,t}, c_{4,t}]$ as interest rate risk factors.

Table 4.4 displays the descriptive statistics of the estimated Svensson factors. Both yield curves for Switzerland and Germany behave similarly. The level of Swiss interest rates, represented by $c_{1,t}$, is on average 0.8 percentage points lower than the German level. The autocorrelation of the coefficients and the cross-correlation between factors is significant.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Switzerland</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{c}_{1,t}$</td>
<td>3.464</td>
<td>4.230</td>
</tr>
<tr>
<td>$\hat{c}_{2,t}$</td>
<td>-2.082</td>
<td>-1.933</td>
</tr>
<tr>
<td>$\hat{c}_{3,t}$</td>
<td>0.048</td>
<td>3.835</td>
</tr>
<tr>
<td>$\hat{c}_{4,t}$</td>
<td>-1.145</td>
<td>-1.112</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>1.084</td>
<td>1.645</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>0.967</td>
<td>0.943</td>
</tr>
<tr>
<td>JB-p</td>
<td>0.042</td>
<td>0.008</td>
</tr>
<tr>
<td>$\hat{c}_{1,t}$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$\hat{c}_{2,t}$</td>
<td>-0.530</td>
<td>-0.396</td>
</tr>
<tr>
<td>$\hat{c}_{3,t}$</td>
<td>-0.537</td>
<td>-0.851</td>
</tr>
<tr>
<td>$\hat{c}_{5,t}$</td>
<td>0.567</td>
<td>0.723</td>
</tr>
</tbody>
</table>

Table 4.4: Estimation results for the Svensson coefficients. Rows 1 to 4 show the mean, the standard deviation, the first sample autocorrelation and the p-value of the Jarque-Bera test statistic for normality of first differences of the coefficients. Rows 5 to 8 show the correlation matrix of the coefficients.

4.4.2 Sampling spot rate distributions

With the constrained Svensson model at hand, we now turn to the question of whether the interest rate distributions resulting from this model in connection with the introduction of plane-truncated error terms can match empirically observed patterns. It is not evident a priori that our methodology will be capable of replicating observed interest rate distributions. Indeed, the ability to produce spot rates that resemble empirical data is merely a question of the appropriateness of the assumed stochastic process underlying $\theta_t$. Furthermore, this question cannot be answered by means of comparison with the in-sample model fit.

In order to assess the stochastic properties of our methodology, we need to simulate monthly changes in the yield curve – and by that we obviously also replicate changes in interest rates for all maturities. In a second step we compare moments of the changes in
the simulated interest rates with the empirically observed moments for each sample path, respectively.

The procedure is straightforward and was essentially stipulated in the model recipe. We first estimate the parameters of the truncated Gaussian $\mu$ and $\Sigma$ by GMM over the whole sample. Having obtained the location and scale parameters, we simulate 10,000 paths of spot rate curves starting in January 2000 up to October 2012. From this procedure, we obtain a matrix of predictions of $\hat{\theta}_t$ that all fulfil the given constraints. It is then possible to evaluate the function of the spot rate term structure (Equation 4.6) for any given maturity and for each path in order to arrive at the distribution of simulated spot rate time series for each maturity. Lastly, we compare the moments of the first differences in simulated spot rates with the moments of the empirically observed spot rate differentials.

Table 4.5 compares observed sample moments of the 5- and 10-year to maturity spot rates with the moments of the corresponding simulated data. For this analysis, we used plane-truncated Gaussian innovations with a lower bound of −50 basis points for instantaneous forward rates ($f(\tau, \hat{\theta}_{t+1}) \geq -0.50$). For each of the 10,000 interest rate paths we calculate the moments presented in the columns of the displayed table. Henceforth, the distribution of the calculated moments is summarised by the percentiles and the means of these simulated moments i.e. the rows of Table 4.5. The average Kurtosis of all draws in our Monte Carlo analysis therefore is 2.989 for 5 years to maturity Swiss spot rates. We then may compare this figure to our in-sample estimate of actual Swiss spot rates that is 3.175. This number is well below the third quartile of the simulated Kurtosis (3.196) and we therefore conclude that our process is able to replicate the empirical observed moment.

An in-depth inspection shows that in January 2000 the interest rate level was fairly high, the constraints were hardly binding at the starting point of the simulations and the innovation was consequently almost Gaussian. This is reflected in the simulated data that show an average skewness close to zero and an average kurtosis of roughly 3. Since none of the sample moments in Table 4.5 are outside the confidence interval we conclude that the risk model is able to generate empirically observable spot rate distributions.

One should recall that the results in Table 4.5 arise from linear combinations of $\theta_t$, rather than directly from simulations of spot rates. It is also important to stress once more that with the methodology at hand any spot rate can be derived. The decision to present the 5-year and 10-year spot rates is somewhat arbitrary; however, it is straightforward to carry out this analysis for further maturities with equivalent results. As indicated in Table 4.5 it suffices to model $\theta_t$ to obtain accurate yield distributions – without simulating...
yields directly. The analysis highlights the flexibility of the proposed approach by showing that it is able to reproduce empirically observed data for interest rates of any maturity.

<table>
<thead>
<tr>
<th></th>
<th>CH 60 months</th>
<th></th>
<th>CH 120 months</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>q1</td>
<td>-0.022</td>
<td>0.140</td>
<td>-0.477</td>
<td>2.313</td>
</tr>
<tr>
<td>q25</td>
<td>-0.017</td>
<td>0.155</td>
<td>-0.141</td>
<td>2.717</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.012</td>
<td>0.162</td>
<td>-0.009</td>
<td>2.989</td>
</tr>
<tr>
<td>q75</td>
<td>-0.008</td>
<td>0.168</td>
<td>0.120</td>
<td>3.196</td>
</tr>
<tr>
<td>q99</td>
<td>0.010</td>
<td>0.185</td>
<td>0.456</td>
<td>4.175</td>
</tr>
<tr>
<td>Sample Est.</td>
<td>-0.021</td>
<td>0.165</td>
<td>-0.248</td>
<td>3.175</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>DE 60 months</th>
<th></th>
<th>DE 120 months</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>q1</td>
<td>-0.035</td>
<td>0.184</td>
<td>-0.466</td>
<td>2.321</td>
</tr>
<tr>
<td>q25</td>
<td>-0.029</td>
<td>0.205</td>
<td>-0.134</td>
<td>2.711</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.020</td>
<td>0.214</td>
<td>-0.006</td>
<td>2.977</td>
</tr>
<tr>
<td>q75</td>
<td>-0.014</td>
<td>0.222</td>
<td>0.124</td>
<td>3.184</td>
</tr>
<tr>
<td>q99</td>
<td>0.012</td>
<td>0.243</td>
<td>0.453</td>
<td>4.155</td>
</tr>
<tr>
<td>Sample Est.</td>
<td>-0.031</td>
<td>0.215</td>
<td>-0.141</td>
<td>2.596</td>
</tr>
</tbody>
</table>

Table 4.5: Comparison of simulated and empirical moments of first differences in spot rates. Rows show the mean, the 1st, 25th, 75th and 99th percentile of the simulated moments. The final row shows the observed sample moment.

### 4.4.3 Comparison with the SST standard model

In the preceding section, we showed that the combination of the constrained Svensson model with plane-truncated Gaussian innovations leads to a risk model that is able to replicate empirically observed moments of spot rate distributions. Nevertheless, it has to be admitted that the results in Table 4.5 could also have been produced using just Gaussian innovations; from this point of view we failed to provide motivation for the usage of a plane truncation. In this section, we address this issue.

The appropriateness of the assumption of truncated innovations is motivated by the firm conviction that a lower bound on nominal interest rates exists. For the analysis executed in this section we applied a plane truncation to the innovations such that the instantaneous forward rates at $\tau = [0, 12, 24, ..., 600]$ were higher than $-50$ basis points. The introduction of a lower bound to Gaussian innovations will alter the shape of the spot rate
distributions significantly. In Subplot (a) of Figure 4.2 we depict twelve-months-ahead predictions of the CHF 1-year to maturity spot rate. The starting point of the prediction is the observed spot rate as at October 2012. In order to compare our model forecasts with those of the SST standard model, we superimpose the density distribution of the SST standard model as a grey line.

In October 2012 the Swiss spot rate level was very low, thus the truncation is highly effective. As a consequence, a large chunk of density mass is moved from the negative towards the positive; the resulting distribution resembles a shifted Log-Normal distribution. Given the truncation constraint, one might have expected a sharp cut of the density distribution at −50 basis points; however, the figure proves this assumption wrong. The resulting density function is rather smooth. The smoothness of the spot rate density has its roots in one distinctive feature of the truncation. Actually, the truncation of the space of \( \theta_t \) acts as a constraint on the instantaneous forward rates rather than directly on the spot rates. As a result, the simulated spot rate density distributions become smooth.

Figure 4.2 gives a graphical impression of the model mechanics. Subplots (a) and (b) depict Swiss interest rates of the two maturities 1 and 10 years. Subplots (b) and (c) show the implications from different interest rate regimes. The only difference between (b) and (c) is the starting point of the simulation. In Subplot (b), the starting yield is the Swiss spot rate as at October 2012 (0.6%), while in Subplot (c) the Swiss interest rate as at January 2000 (3.8%) was used. When we compare our model with the SST standard model in Subplot (b), we find that the expectation of the spot rate in \( t+12 \) implied by our model is shifted to the right. Even more interestingly, the model implied expectation in Subplot (c) is shifted to the left when compared with a pure Gaussian without truncation. This graphically highlights the point made in Section 4.2.3.3: We argue that, depending on the parameter estimates of \( \mu \) and \( \Sigma \), a non-stationary truncated Gaussian process leads to mean-reverting spot rates. As it happens, the parameter estimator suggests that spot rates are actually mean-reverting.

The economic implications of the introduction of truncated Gaussians are of fundamental importance for insurers. As set out in Section 4.1, the SST standard model sets interest rate distributions disregarding the current state of the economy. This results in target capital requirements that are independent of the state of the economy. The model we suggest adapts to the state of the economy. Compared to our findings, the SST standard model results in lower solvency capital requirements during high yield phases where downward potential is high. At the same time, the standard model leads to relatively higher capital requirements in low yield regimes when interest rate downward potential approaches zero due to the lower bound on interest rates. From a macroeconomic per-
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spective, the SST standard model may cause insurers to potentially over-invest in boom phases and hold back funds during bust environments. This pattern is known as procyclicality. For an early study in the banking sector, see for example Blum and Hellwig (1995). The proposed model, however, introduces a countercyclical momentum since capital requirements correspond to the prevailing interest rate environment. Firstly, interest rate distributions are skewed in a low interest rate environment and rather symmetric when the prevailing interest rate level is high. Secondly, the centre of the distribution converges towards its long-run mean. Comparison of Subplot (b) and Subplot (c) highlights the countercyclicality of the model.

Figure 4.2: Comparison of CHF spot rate density distributions. The black area illustrates the simulated density of the discussed model. The grey line shows the yield density distribution of the SST standard model. Subplots (a) and (b) depict the interest rates distributions for 1 and 10 years to maturity for \( t + 12 \) as at reporting date October 2012. Subplot (c) shows the 10 year to maturity interest rate distribution for \( t + 12 \) as at January 2000.

4.5 Conclusion

The Swiss Solvency Test requires insurers develop a market-consistent valuation of assets and liabilities. The calculation of solvency capital requirements builds on market-consistent valuation as well. Since market values of insurance liabilities are very rare, interest rates are key for the valuation of liabilities and for the calculation of capital requirements from asset-liability mismatch.

This paper analyses the treatment of interest rate risk under the SST with the standard model and finds three essential shortcomings. Firstly, the standard risk model suggests a considerable number of interest rate risk factors. The risk manager is confronted with
the complex and laborious task of modelling and numerically simulating thirteen interest rate buckets – for each currency. Such a requirement becomes even more futile when the data generating process of spot rate yield curves is considered. Secondly, under the SST, changes in spot rates are assumed to be Gaussian. This allows for highly negative interest rates – especially in low interest rate environments such as those faced during the ongoing financial and economic crisis. Thirdly, and most importantly, in the case of a positive interest rate sensitivity of the Risk Bearing Capital, the utilization of Gaussians automatically introduces procyclical capital requirements. The standard model forces the insurance undertaking to hold more solvency capital in a low interest rate environment even though the downward interest rate risks is less pronounced than usual.

The new methodology concentrates on modelling interest rates and addresses the flaws of the SST. Firstly, a systematic analysis of the term structure of spot rates suggests a reasonable simplification of the risk model. By explicitly considering of the data generating process for spot rate curves, the number of risk factors can be significantly reduced to the four factors of the Svensson model. Secondly, the stochastic process used to model spot rates is built around a truncated Gaussian which allows for the introduction of a spot rate floor. The existence of a floor is reasonable from a theoretical point of view since negative interest rates should at most reflect the cost of holding cash. This is also suggested from an empirical analysis as highly negative interest rates have never been observed. Using a Method of Moments-type estimator, the truncated Gaussian process results in a mean-reverting interest rate process that matches empirical observations. Thirdly, the suggested model yields a distribution of interest rates that adapts to the prevailing economic regime. In low interest rate environments, the distribution is positively skewed while in high interest rate environments the spot rate distribution is symmetric. Finally, the paper puts forward a model recipe for implementation and delivers an empirical discussion of the results obtained.

Appendix

The mean-reverting behaviour of a truncated Gaussian process

In order to show that the process

\[ r_{t+1} = r_t + \epsilon_t, \quad \epsilon_t \sim TN(\mu, \sigma, | \Omega_t) \quad \text{where} \quad \Omega_t = \{ \epsilon_t | \epsilon_t \geq a - r_t = a_t \} \] (4.25)

is converging towards its long-run mean \( r^* \), we have to show that
Interest rate risk and the Swiss Solvency Test

1. the long-run mean \( r^* \) exists and
2. the random number \( r_t \) is pulled towards \( r^* \).

We start exploring the restrictions on \( \mu \) and \( \sigma \) such that \( r^* \) exists. The conditional expectation of the truncated Gaussian distribution is determined by

\[
E[\epsilon_t \mid \Omega_t] = \frac{1}{\alpha_t} \int_{a_t}^{\infty} \epsilon_t N(\mu, \sigma) d\epsilon_t = \mu + \sigma \cdot \lambda(\beta_t),
\]

where \( \lambda(\beta_t) \) is the inverse Mills ratio,

\[
\lambda(\beta_t) = \frac{\phi(\beta_t)}{1 - \Phi(\beta_t)}.
\]

\( \phi \) and \( \Phi \) denote the standard normal density and cumulative density distribution respectively; \( \beta_t = \frac{a_t - \mu}{\sigma} \). As several authors point out, the inverse Mills ratio is monotonically increasing and continuous in \( \beta_t \), consequently \( E[\epsilon_t \mid \Omega_t] \) is continuous and monotonically increasing in \( \beta_t \) and \( a_t \) (see Hayashi, 2000 for further reference).

Our strategy is to calculate \( E[\epsilon_t \mid \Omega_t] \) at the lower and upper bound of \( a_t \). If \( E[\epsilon_t \mid \Omega_t] \) is negative at the lower bound and positive at the upper bound the existence of \( E[\epsilon_t \mid \Omega_t] = 0 \) follows from continuity of \( E[\epsilon_t \mid \Omega_t] \) in \( a_t \). The lower bound of \( a_t \) is \( a_t = -\infty \) and the upper bound is given by \( a_t = 0 - \infty \) and the upper bound is given by \( a_t = 0 \) – in this case today’s observation is exactly at the bound \( a = r_t \). Note that

\[
\lim_{a_t \to -\infty} E[\epsilon_t \mid \Omega_t] = \mu.
\]

Since \( E[\epsilon_t \mid \Omega_t] \) has to be negative at the lower bound it follows that \( \mu \) has to be negative. \( E[\epsilon_t \mid \Omega_t] \) at the upper bound is given by

\[
E[\epsilon_t \mid \Omega_t] = \mu + \sigma \cdot \lambda \left( \frac{0 - \mu}{\sigma} \right).
\]

Thus, \( r^* \) exists, if and only if

\[
\mu \leq 0 \leq \mu + \sigma \cdot \lambda \left( \frac{0 - \mu}{\sigma} \right). \tag{4.26}
\]

Given a set \( \mu \) and \( \sigma \) that fulfils Equation 4.26 we finally have to show that \( r_t \) is pulled to \( r^* \). This is achieved by demonstrating that the expectation of \( \epsilon_t \) is positive if today’s

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32 It should be noted that at \( r_t = r^* \) the conditional expectation of \( \epsilon_t \) is \( E[\epsilon_t \mid \Omega_t] = 0 \).

33 Again, we condition on the information set at time \( t \). It immediately follows that \( \beta_t \) and \( a_t \) are given. Furthermore, note that \( N(0,1) = \phi(\beta_t) \), whereas \( \Phi(\beta_t) = \int_{-\infty}^{\beta_t} N(0,1)dx \).
level of $r_t$ is below $r^*$ and negative if $r_t$ is above $r^*$. We start by showing that if $r_t < r^*$, 
$E[\epsilon_t | \Omega_t] > 0$.

$$a^* = a - r^* < a - r_t = a^{**}$$

(4.27)

Since $E[\epsilon_t | \Omega_t]$ is monotonically increasing in $a_t$ and $E[\epsilon_t | \Omega_t] = 0$ at $a^*$, it follows that 
$E[\epsilon_t | \Omega_t] > 0$ at $a^{**}$. The converse holds true for $r_t > r^*$ establishing that the truncated 
Gaussian process defined in Equation 4.25 is mean-reverting if $r^*$ exists.
5 Concluding remarks and outlook

It is key for economists to analyse and understand economic crises in detail in order to develop solid economic theory and improve (financial) regulation. In this thesis, we analyse financial stability by investigating three sources of risk: liquidity risk, contagion risk and interest rate risk.

In chapter 2 we show that a possible future liquidity shock may erode financial stability and potentially causes asset price bubbles. Against this background internationally harmonized liquidity standards may be desirable. Moreover, central bank interventions could stabilize the financial system by injecting liquidity in the financial system and this might reduce asset price bubbles as well. However, regulatory liquidity requirements and central bank interventions are not explicitly considered in this model. Thus, it is not possible to analyse the efficiency of liquidity regulation and central bank interventions in detail.

The question about the importance of spill overs across financial institutions is the starting point of the second essay. The econometric approach chosen stems from spatial econometrics. The definition of contiguity is re-interpreted and provides a valuable means of analysis. We find a considerable amount of risk propagation within the system. Nevertheless, the effect’s origin still remains ambiguous. On the one hand, the increased risk charge may evolve from counterparty risk since CDS contracts are only written by a few issuers. On the other hand, the risk charge might stem from the interconnectedness of financial institutions and hence from contagion risk. In any case, the risk charge is highly relevant and present within the system. The identification problem of the origin of the risk charge can be overcome by analysing bond spreads with a similar model since fixed income markets are deeper and broader. Moreover, this would increase the sample size since almost every financial institutions has issued a fixed income instrument in the past but not on many institutions CDS contracts are written. This holds especially true for cooperative banks which hold a huge share in the German and Austrian market.

In chapter 4 the essay deals with the regulatory framework for insurers within Switzerland. The analysis specifically lays its focus on the data generating process of interest rates and develops a solid framework for obtaining real world interest rate distributions.
The analysis of the currently applicable standard model shows that the separate modelling of interest rates for 13 maturities is futile given the data generating process of spot rate curves. Furthermore, a novel interest rate sampling mechanism is proposed that specifically addresses the zero lower bound restriction of interest rates. With this procedure at hand, it is possible to generate real world yield curves. Thus, the model is particularly suited for risk management purposes. Nonetheless, actuaries have to calculate the best estimate of liabilities which is typically done by arbitrage free pricing and risk neutral simulation. Thus, it would be very interesting to transform the real world model into a risk neutral model. This would imply transforming the probability measure and this was beyond the scope of the analysis.

In comparison with the SST standard model, the proposed model introduces several layers of complexity. Thus, it is very well justified to doubt the usefulness, efficiency and adequacy of such complex models. However, the gains of the proposed model framework are manifold. Firstly, the parameter space to model interest rate risk is substantially reduced. This saves time and money when it comes to computational burdensome RBC sensitivity analyses. Secondly, in low interest rate environments the regulatory capital is significantly reduced; hence the firm's capital cost will decline. Finally, a regulator is very well advised to implement a countercyclical policy and a model which adapts to the prevailing economic environment. This is achieved by implementing a more complex model. All in all we are convinced that it is the effort worth to implement a more complex model.

To summarise, in this thesis we analyse financial stability and regulation with rather different research methods - microeconomic modelling, empirical research and random process modelling - and propose several instruments to improve financial regulation. Our findings should contribute to a sound and save financial market.

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1 Let us assume that an average stochastic liability model has a run-time of 10 minutes. In order to parametrise the SST standard model valuation function with respect interest rate risk 52 times 2 sensitivity runs are necessary. Additional $52 \cdot 51 \cdot 4 \cdot 0.5 = 5304$ cross sensitivity valuation runs have to be executed. This leads to a run-time of 54060 minutes which are 38 days.


Bibliography


Bibliography


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