Barodesy for clay

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Barodesy is a new hypoplastic frame for constitutive models to describe granular materials. It has already been introduced for sand, and this article adjusts barodesy for clay. Common concepts of soil mechanics, such as critical states, barotropy (i.e. the dependence of stiffness and strength on the stress level), pyknotropy (i.e. the dependence of stiffness and strength on density) and a stress–dilatancy relation can easily be included in the presented model. Despite its mathematical simplicity, barodesy is able to describe many aspects of clay behaviour. Only four material constants are required, which can be determined by means of a single consolidated undrained triaxial test.

KEYWORDS: clays; constitutive relations

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INTRODUCTION

A new constitutive frame for soil called ‘barodesy’, proposed by Kolymbas (2009, 2011, 2012) and conceived as an evolution equation of the type $\dot{\mathbf{T}} = \mathbf{h}(\mathbf{T}, \mathbf{D}, e)$, is based on the asymptotic behaviour of granulates. It can also be considered as a further development of hypoplasticity (Kolymbas, 1977, 1985, 1991, 2000). Barodesy is derived from two rules proposed by Goldscheider (1976), based on the results of true triaxial tests. He applied proportional strain paths to sand samples and observed the corresponding stress paths. Proportional strain paths are paths with constant ratios of the principal strains (i.e. $\varepsilon_1 : \varepsilon_2 : \varepsilon_3 = \text{constant}$). In the same sense, paths with constant ratios of the principal stresses are called proportional stress paths (i.e. $\sigma_1 : \sigma_2 : \sigma_3 = \text{constant}$). Goldscheider (1976) formulated the following two rules.

- Rule 1. Starting at a stress-free state $\mathbf{T} = \mathbf{0}$, proportional strain paths lead to proportional stress paths (see Fig. 1(a) and Fig. 1(b)).
- Rule 2. Starting at $\mathbf{T} \neq \mathbf{0}$, proportional strain paths lead asymptotically to the corresponding proportional stress paths starting at $\mathbf{T} = \mathbf{0}$ (see Fig. 1(a) and Fig. 1(c)). This means that proportional stress paths act as attractors.

SYMBOLS AND NOTATION

We adhere to the symbolic notation of stress with $\mathbf{T}$ but, in some cases, we use for the principal stresses the more familiar symbol σ_i instead of T_i . The stretching tensor $\mathbf{D}$ (i.e. the symmetric part of the velocity gradient) corresponds approximately to the strain rate, but is strictly speaking not the time rate of any strain measure. Superscript 0 denotes a normalised tensor (i.e. $\mathbf{X}^0 = \mathbf{X}/|\mathbf{X}|$ with $|\mathbf{X}| = (\text{tr } \mathbf{X}^2)^{1/2}$). The following abbreviations are used: $\epsilon := \text{tr } \mathbf{D}^0$, $\sigma := |\mathbf{T}|$, $\dot{\epsilon} := |\dot{\mathbf{D}}|$. Stresses should be understood as effective ones; the normally used dash is omitted here for brevity, even for $p := -(1/3) \text{tr } \mathbf{T}$. As in this article only rectilinear extensions are examined, the co-rotational stress rate $\dot{\mathbf{T}}$ is replaced with $\dot{\mathbf{T}}$. The void ratio e is, as usual, the ratio of the volume of the voids V_p to the

volume of the solids V_s . p is the mean effective stress and $\epsilon_v = \text{tr } \mathbf{\epsilon}$ is the volumetric strain. For a conventional triaxial compression test, the axial stress is denoted with σ_1 and the radial stress is denoted with $\sigma_2 (= \sigma_3)$. The associated strains are ϵ_1 and $\epsilon_2 = \epsilon_3$. Recall that in critical state soil mechanics (CS SM), the variables $q := \sigma_1 - \sigma_3$ and $\epsilon_q := 2/3 \cdot (\epsilon_1 - \epsilon_3)$ are used.

CONSTITUTIVE EQUATION

The directions $\mathbf{R}$ of proportional stress paths starting at $\mathbf{T} = \mathbf{0}$ and obtained with proportional strain paths $\mathbf{D} = \text{constant}$ are given by (Kolymbas, 2009)

$$\mathbf{R}(\mathbf{D}) = (\text{tr } \mathbf{D}^0) \mathbf{1} + c_1 \exp(c_2 \mathbf{D}^0) \quad (1)$$

where c_1 and c_2 are constants.¹

The general form of the barodetic constitutive relation is (Kolymbas, 2011)

$$\dot{\mathbf{T}} = h(\sigma) \cdot (f \mathbf{R}^0 + g \mathbf{T}^0) \cdot \dot{\epsilon} \quad (2)$$

with $h = c_3 \sigma^{c_4}$, where c_3 and c_4 are material constants.

In this article, a new formulation of the functions f and g (e.g. for sand, it can be set $f = c_5 \epsilon - e_c$ and $g = e$) to model clay behaviour is used

$$f = c_5 \epsilon - \ln(1 + e) \quad (3)$$

$$g = \ln(1 + e) \quad (4)$$

The scalar functions f and g take into account critical states, barotropy and pyknotropy. How f and g are derived, based on concepts of CSSM, is shown in the following section. The quantity e_c is the stress-dependent critical void ratio (see equation (6) and c_5 is a material constant. Equations (1)–(4) are the full set of barodetic equations for clay. Note that the notation of the material constants c_1 – c_5 is chosen according to Kolymbas (2011).

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¹Note that the exponential of a tensor $\mathbf{A}$ can be defined by means of its eigenvalues a_i

$$\exp \mathbf{A} = \exp \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix} = \begin{pmatrix} \exp a_1 & 0 & 0 \\ 0 & \exp a_2 & 0 \\ 0 & 0 & \exp a_3 \end{pmatrix}$$

Fig. 1. (a) Proportional strain paths. (b) Proportional stress paths. (c) Asymptotic behaviour

LINKS TO CRITICAL STATE SOIL MECHANICS

We start from two well-known equations from CSSM.

- The isotropic normal consolidation line (NCL) (Butterfield, 1979)

$$\ln(1+e) = N - \lambda \ln \frac{p}{p_r} \quad (5)$$

in which N is the ordinate intercept (i.e. $N = \ln(1+e)$ at $p = p_r = 1$ kPa) and λ is the slope of the NCL (see Fig. 2(a)). (Note that, usually, the quantities N and λ are defined in a $\ln p - (1+e)$ plot, whereas the symbols N^* and λ^* are used with reference to $\ln p - \ln(1+e)$ plots. In this article, we write N and λ for brevity instead of N^* and λ^* .)

- The critical state line (CSL) is defined as follows and is parallel to the NCL

$$\ln(1+e_c) = N - \lambda \ln \frac{2p}{p_r} \quad (6)$$

Note that according to the so-called ‘state boundary surface’ of the modified Cam clay model (MCCM) (Roscoe & Burland, 1968) the ordinate intercepts of the CSL and NCL are related (see Fig. 2(a)).

Critical states are characterised by a vanishing stress rate $\dot{\mathbf{T}} = \mathbf{0}$ and isochoric deformation $\epsilon = 0$. According to equation (2), the condition $\dot{\mathbf{T}} = \mathbf{0}$ implies that $\mathbf{R}^0 = \mathbf{T}^0$ and $f + g = 0$. From equations (2) and (6), we get, for critical states ($e = e_c$ and $\epsilon = 0$)

$$f + g = 0 = \ln(1+e_c) - N + \lambda \ln \frac{2p}{p_r} \quad (7)$$

The following generalisation for non-isochoric deformations ($e \neq e_c$, $\epsilon \neq 0$) is assumed

$$f + g = \ln(1+e) - N + \lambda \ln \frac{2p}{p_r} + c_5 \epsilon \quad (8)$$

With equation (6), it follows that

$$f + g = \ln(1+e) - \ln(1+e_c) + c_5 \epsilon \quad (9)$$

The following partition of $f + g$ is chosen:

$$f = c_5 \epsilon - \ln(1+e_c) \quad (10)$$

$$g = \ln(1+e) \quad (11)$$

It should be mentioned that the critical state surface (Fig. 3(b)) depends only on the critical friction angle φ_c and practically coincides with the Matsuoka–Nakai failure criterion (Fellin & Ostermann, 2011) (Fig. 3(a)).

CALIBRATION

Experimental results with Dresden clay were used for the calibration (Dresden clay data are taken from Bergholz (2009)). For determination of the four parameters of CSSM (N , λ , φ_c and $\hat{G}_0$), an isotropic compression test and an undrained triaxial test are sufficient. Both are performed in the course of a consolidated undrained (CU) triaxial test.

The slope λ and the intercept N of the NCL are obtained from the isotropic compression (Fig. 2). The critical friction angle φ_c can be determined, for example, by an undrained triaxial compression test (Fig. 4). The slope of the CSL in the p - q plane is

$$\frac{q_c}{p_c} = \frac{6 \sin \varphi_c}{3 - \sin \varphi_c}$$

From a critical stress state, φ_c can be obtained

$$\sin \varphi_c = \frac{3q_c}{q_c + 6p_c} \quad (12)$$

Fig. 2. Normal compression line (NCL) and critical state line (CSL). (a) Schematic diagram. (b) Isotropic compression: experimental results with Dresden clay CD 02 and numerical simulation with barodesy, $\ln(1+e)$ versus $\ln p$ plot

Fig. 3. Critical state surface obtained with equation (1). (a) Matsuoka-Nakai failure surface and critical state surface obtained with barodesy. (b) Critical state surface according to barodesy

Now we consider the initial stiffness of an undrained triaxial compression test. It is common to express the stiffness $\partial q / \partial \varepsilon_q$ by means of the shear modulus G

$$G = \frac{1}{3} \frac{\partial q}{\partial \varepsilon_q} \quad (13)$$

The initial value for G is $G_0 = G|_{\varepsilon_q=0}$ (see Fig. 5(a)). Normalising the stiffness G with the consolidation pressure p_0 (see Fig. 4(a)) and with the critical void ratio at this pressure $e_c(p_0)$ yields

$$\hat{G} := \frac{G}{p_0 \cdot e_c(p_0)} \quad (14)$$

It can be seen in Fig. 5(b), that the dependence of $\hat{G}$ on ε_q unifies undrained triaxial test results obtained with various p_0 values. The influence of $\hat{G}_0 := \hat{G}|_{\varepsilon_q=0}$ on the simulations of drained triaxial tests with barodesy is rather small (see Fig. 6).

The method of calibrating the constants c_1 – c_5 on the basis of the aforementioned parameters φ_c , N , λ and $\hat{G}_0$ is now described.

Calibration of c_2

At critical states, the stress rate vanishes, $\dot{\mathbf{T}} = \mathbf{0}$. Consequently, $\mathbf{R}$ is proportional to $\mathbf{T}$, i.e. $\mathbf{R}^0 = \mathbf{T}^0$ (see equation (2)). For an undrained triaxial test, the dilatancy vanishes, i.e. $\epsilon = 0$ and

the asymptotic stress ratio is given by the critical friction angle φ_c (Fig. 4). With

$$K_c := \frac{1 - \sin \varphi_c}{1 + \sin \varphi_c} = \frac{\sigma_2}{\sigma_1} = \frac{R_2}{R_1} = \exp(c_2 D_2^0 - c_2 D_1^0)$$

it follows that

$$c_2 = \frac{\ln K_c}{D_2^0 - D_1^0} = \left(\frac{2}{3}\right)^{1/2} \ln K_c = \left(\frac{2}{3}\right)^{1/2} \ln \frac{1 - \sin \varphi_c}{1 + \sin \varphi_c} \quad (15)$$

Calibration of c_1

We consider the peak of a drained triaxial (CD) test with $\dot{\mathbf{T}} = \mathbf{0}$ and

$$K_a := \frac{\sigma_2}{\sigma_1} = \frac{1 - \sin \varphi_p}{1 + \sin \varphi_p}.$$

Fig. 4. Calibration of φ_c on the basis of CU tests. The experimental results in (b) (CU 03, CU 04 and CU 05) were obtained with Dresden clay

Fig. 5. Calibration of $\hat{G}_0 := G_0 / (p_0 \cdot e_c(p_0))$ with $G_0 = G|_{\varepsilon_q=0}$ on the basis of CU tests. The experimental results (CU 03, CU 04 and CU 05) were obtained with Dresden clay

Fig. 6. Influence of the initial normalised stiffness $\hat{G}_0$ on numerical simulations of CD tests ($\sigma_3 = 200$ kPa) with barodesy (other material parameters as in Table 1)

$\dot{\mathbf{T}}$ vanishes in equation (2) if $\mathbf{R}^0 = \mathbf{T}^0$ and $f + g = 0$. Therefore we set

$$\frac{R_2}{R_1} = \frac{\sigma_2}{\sigma_1}$$

The stretching at the peak

$$\mathbf{D} = \begin{pmatrix} D_1 & 0 & 0 \\ 0 & D_2 & 0 \\ 0 & 0 & D_2 \end{pmatrix}$$

can be evaluated as follows (see Fig. 7(a))

$$\begin{aligned} \tan \beta &= \frac{-\dot{\epsilon}_v}{\dot{\epsilon}_1} = \frac{-\text{tr } \mathbf{D}}{D_1} = -\frac{D_1 + 2D_2}{D_1} \\ &= -1 - \frac{2D_2}{D_1} \rightsquigarrow D_2 = \frac{-D_1(\tan \beta + 1)}{2} \end{aligned}$$

From the ratio D_2/D_1 , the dilatancy measure $\epsilon = \text{tr } \mathbf{D}^0$ can be determined. With $\sigma_2/\sigma_1 = K_a$, we obtain

$$\frac{R_2}{R_1} = \frac{\epsilon + c_1 \exp(c_2 D_2^0)}{\epsilon + c_1 \exp(c_2 D_1^0)} = K_a$$

and then

$$c_1 = \frac{\epsilon(1 - K_a)}{K_a \exp(c_2 D_1^0) - \exp(c_2 D_2^0)} \quad (16)$$

With an empirical relationship between certain stress ratios and dilatancy $\tan \beta = -\dot{\epsilon}_v/\dot{\epsilon}_1$, according to Chu & Lo (1994) and Luo *et al.* (2012), determination of c_1 (equation (16)) can be simplified. Let us consider states characterised with $\mathbf{R}^0 = \mathbf{T}^0$. Such states are (i) proportional paths (e.g. isotropic compression, critical states) and (ii) peak states. Chu & Lo (1994) observed that the corresponding stress

ratios $\eta_a = q/p$ of such states are related to the dilatancy $\tan \beta$ as shown in Fig. 7. For isotropic compression we have $\tan \beta = -3$ and $\eta_a = 0$; for undrained triaxial compression $\tan \beta = 0$ and, at critical states, $\eta_a = q_c/p_c = 6 \sin \varphi_c / (3 - \sin \varphi_c)$. Connecting the states of isotropic compression and the critical state in the η_a - $\tan \beta$ plot leads to the following linear relation between η_a and $\tan \beta$

$$\eta_a = \frac{q}{p} = \frac{M}{3} \tan \beta + M \quad \text{with } M = \frac{6 \sin \varphi_c}{3 - \sin \varphi_c} \quad (17)$$

It can be seen from Fig. 7(b) that peak states from drained triaxial (CD) tests can be approximated by equation (17) (i.e. by means of critical friction angle and dilatancy).

We replace K_a with $(3 - \eta_a)/(3 + 2\eta_a)$ in equation (16) and use η_a from equation (17) with $\tan \beta = 1$ (i.e. $D_{22} = D_{33} = 1$ for $D_{11} = -1$) to rewrite equation (16) as a function of φ_c

$$c_1 = \frac{-8 \times 3^{1/2} \sin \varphi_c}{(13 \sin \varphi_c + 9) K_c^{\sqrt{2}/3} + (11 \sin \varphi_c - 9)(1/K_c)^{\sqrt{2}/3}} \quad (18)$$

Calibration of c_4 and c_5

Equation (5) is used for the calibration of c_4 and c_5 to achieve that, at isotropic normal compression, barodesy complies with the NCL (see similar approaches by Mašin (2005, 2012)). Derivation of equation (5) with respect to time t gives

$$\frac{\dot{\epsilon}}{1+e} = -\lambda \frac{\dot{p}}{p} \quad (19)$$

With $\dot{\epsilon}/(1+e) = \text{tr } \mathbf{D} = -3^{1/2} |\mathbf{D}| = -3^{1/2} \dot{\epsilon}$ (the equation $\dot{\epsilon}/(1+e) = \text{tr } \mathbf{D}$ holds for incompressible grains) and $\sigma = 3^{1/2} p$, the NCL reads

Fig. 7. Determination of c_1 based on peak states of CD tests. The experimental results (CU and CD triaxial tests) refer to Dresden clay. (a) CD test: schematic diagram. (b) Stress ratio $\eta_a = p/q$ versus dilatancy, according to Chu & Lo (1994) and Luo *et al.* (2012)

Table 1. CSSM parameters used for calibration of barodesy

Material	φ_c : degrees	N	λ	$\hat{G}_0$
London clay	22.6	1.375	0.110	58
Dresden clay	35.0	0.622	0.038	150

Table 2. Barodesy for clay. Full set of constitutive equations with the abbreviations $\mathbf{X}^0 = \mathbf{X}/|\mathbf{X}|$, $\sigma = |\mathbf{T}|$, $p = -\text{tr } \mathbf{T}/3$, $\dot{\epsilon} = |\mathbf{D}|$, $\epsilon = \text{tr } \mathbf{D}^0$

Direction of proportional paths	$\mathbf{R}(\mathbf{D}) = \epsilon \mathbf{1} + c_1 \exp(c_2 \mathbf{D}^0)$
Constitutive equation	$\dot{\mathbf{T}} = c_3 \sigma^{c_4} (f \mathbf{R}^0 + g \mathbf{T}^0) : \dot{\epsilon}$ $f = c_5 \epsilon - \ln(1 + e_c)$ $g = \ln(1 + e)$
Critical void ratio	$e_c = \exp[N - \lambda \ln(2p)] - 1$

$$\dot{p} = \sigma \frac{1}{\lambda} \dot{\epsilon} \quad (20)$$

With the general form of the barodetic constitutive relation (equation (2)), isotropic compression (i.e. $\mathbf{T} = -p\mathbf{1}$, $\mathbf{R}^0 = \mathbf{T}^0 = -(1/3^{1/2})\mathbf{1}$) yields

$$\dot{p} = c_3 \sigma^{c_4} \frac{f+g}{3^{1/2}} \dot{\epsilon} \quad (21)$$

Comparing equations (20) and (21) yields

$$c_4 = 1 \quad (22)$$

and

$$\frac{c_3}{3^{1/2}} (f+g) = \frac{1}{\lambda} \quad (23)$$

Now we write equation (8) for hydrostatic compression ($\epsilon = -3^{1/2}$), use equation (23) and obtain

$$c_3 \left(\ln(1+e) - N + \lambda \ln \frac{2p}{p_r} - c_5 3^{1/2} \right) = \frac{3^{1/2}}{\lambda} \quad (24)$$

Introducing equation (5) into equation (24) leads to

$$c_5 = \frac{c_3 \lambda^2 \ln 2 - 3^{1/2}}{c_3 \lambda 3^{1/2}} \quad (25)$$

Equation (25) ensures that a simulation of hydrostatic normal compression with barodesy starting from $e = \exp N - 1$ leads to the NCL (see Fig. 2(b)). Note that $c_4 = 1$ for clay, whereas for sand $c_4 < 1$ (Kolymbas, 2011).

Table 3. Constants c_1 – c_5

$c_1 = \frac{-8 \times 3^{1/2} \sin \varphi_c}{(13 \sin \varphi_c + 9)[(1 - \sin \varphi_c)/(1 + \sin \varphi_c)]^{\sqrt{2}/3} + (11 \sin \varphi_c - 9)[(1 + \sin \varphi_c)/(1 - \sin \varphi_c)]^{\sqrt{2}/3}}$
$c_2 = \left(\frac{2}{3}\right)^{1/2} \ln \frac{1 - \sin \varphi_c}{1 + \sin \varphi_c}$
$c_3 = -\frac{\hat{G}_0 (3 \sin^2 \varphi_c - 2 \sin \varphi_c + 3)^{1/2}}{2^{1/2} \sin \varphi_c}$
$c_4 = 1$
$c_5 = \frac{c_3 \lambda^2 \ln 2 - 3^{1/2}}{c_3 \lambda 3^{1/2}}$

Calibration of c_3

For an undrained test, $\text{tr } \mathbf{D} = 0$ (i.e. $\epsilon_v = \epsilon_1 + 2\epsilon_2 = 0$) and therefore $\epsilon_q = (2/3)(\epsilon_1 - \epsilon_2) = \epsilon_1$. Hence, equation (13), for isochoric deformation, reads

$$G = \frac{1}{3} \frac{\partial q}{\partial \epsilon_1} = \frac{1}{3} \frac{\dot{T}_1 - \dot{T}_2}{D_1} \quad (26)$$

From equations (1), (2) and (22) it follows that

$$G = \frac{\dot{T}_1 - \dot{T}_2}{3D_1} = \frac{c_3 \sigma \dot{\epsilon} (fR_1^0 + gT_1^0 - fR_2^0 - gT_2^0)}{3D_1} \quad (27)$$

The shearing in a CU test starts from a hydrostatic stress state $\mathbf{T} = p_0 \mathbf{1}$ (i.e. $p_0 = T_1 = T_2 = T_3$). Introducing f from equation (10), $\mathbf{R}$ from equation (1), $\text{tr } \mathbf{D} = 0$,

$\mathbf{D} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{pmatrix} \cdot D_1$ and $G_0 = G|_{\epsilon_q=0}$ into equation (27) yields

$$c_3 = -\frac{G_0 (3 \sin^2 \varphi_c - 2 \sin \varphi_c + 3)^{1/2}}{2^{1/2} \sin \varphi_c [N - \lambda \ln(2p_0)] p_0} \quad (28)$$

and, with equation (14), this yields

$$c_3 = -\frac{\hat{G}_0 (3 \sin^2 \varphi_c - 2 \sin \varphi_c + 3)^{1/2}}{2^{1/2} \sin \varphi_c} \quad (29)$$

Note that all parameters N , λ , φ_c and $\hat{G}_0$ can be obtained from one CU triaxial test. Table 1 summarises the material parameters for the considered clays. Table 2 sums up all the equations of barodesy for clay. Determination of the constants c_1 – c_5 is summarised in Table 3.

PROPERTIES OF THE MODEL

To give an idea of the properties of the model, fundamental features are shown. The asymptotic behaviour to critical states is shown in Fig. 8. London clay is simulated with barodesy (Fig. 8) and with Mašin's hypoplastic constitutive model (Mašin, 2005) (Fig. 9). Simulated paths of drained triaxial tests are shown as p – e and p – q plots as well as plots in the normalised stress plane (i.e. p/p_e – q/p_e), where p_e is Hvorslev's equivalent consolidation pressure on the NCL, given by

$$p_e = \exp \left(\frac{N - \ln(1+e)}{\lambda} \right)$$

Fig. 8. Barodesy: simulated paths of drained triaxial tests for London clay. The start points are denoted by a circle (o) and the end-points by a cross (+)

Fig. 9. Hypoplasticity version of Mašin: simulated paths of drained triaxial tests for London clay. The start points are denoted by a circle (o) and the end-points by a cross (+)

Fig. 10. Oedometric compression. (a) Jaky's equation ($K_0 = 1 - \sin \varphi_c$) compared with predictions from barodesy, the MCCM and the hypoplastic version of Mašin; MCCM and Mašin data digitalised from Mašin & Herle (2005). (b) Simulations with barodesy starting from various initial densities

Figure 10(a) shows K_0 predictions of different models compared with Jaky's equation ($K_0 = 1 - \sin \varphi_c$). As reported by Mašin & Herle (2005), based on experimental data (Mayne & Kulhawy, 1982), Jaky's prediction for K_0 is suitable for clay. The K_0 predictions of the MCCM and the hypoplastic version of Mašin were taken from Mašin & Herle (2005).

Figure 10(b) shows the asymptotic behaviour under oedometric compression. The uniaxial compression tests start from various initial densities. Further information of the performance of the model is given in the next section.

SIMULATION OF ELEMENT TESTS

Numerical simulations are compared with experiments and with Mašin's hypoplastic constitutive model for clay (Mašin, 2005), see Table 4 and with the MCCM (Roscoe & Burland, 1968), see Table 5. The MCCM was applied according to Butterfield (1979) (i.e. the NCL is assumed as linear in a log-log plot). The parameters N , λ , κ and φ_c of the MCCM correspond to the parameters of the hypoplastic constitutive model for clay of Mašin (2005) and the initial shear stiffness G_0 is chosen according to Mašin (2004). Note that in Tables 4 and 5, the parameters λ and κ correspond to those denoted by Mašin (2005) as λ^* and κ^* .

Table 4. CSSM parameters used for calibration of Mašin (2005)

Material	φ_c : degrees	N	λ	κ	r
London clay	22.6	1.375	0.11	0.016	0.4

Table 5. CSSM parameters used for calibration of the MCCM, according to Butterfield (1979)

Material	φ_c : degrees	N	λ	κ	G_0 : MPa
London clay	22.6	1.375	0.11	0.016	5

Table 6. Clay characteristics

Material	Major clay mineral type	Proportion of clay: %	w_p : %	w_l : %	I_p : %
Dresden clay	Muscovite/illite	21	19	34	16.5
London clay (Mašin, 2004)	Illite	53	20–30	60–80	40.0

Table 7. Simulations of element tests and experimental results shown in this article

Test		Material	Model
Isotropic compression	Fig. 2(b)	Dresden clay	Barodesy
Triaxial CD	Fig. 6	Dresden clay	Barodesy
Triaxial CD, CU	Fig. 7(b)	Dresden clay	Barodesy
Triaxial CD	Fig. 8	London clay	Barodesy
Triaxial CD	Fig. 9	London clay	Mašin (2005)
Oedometric compression	Fig. 10(b)	London clay	Barodesy
Triaxial CD	Fig. 11(a)	London clay	Barodesy (Mašin, 2005)
$p = \text{constant}$			MCCM (Roscoe & Burland, 1968)
Isotropic compression	Fig. 11(b)	London clay	Barodesy (Mašin, 2005)
			MCCM (Roscoe & Burland, 1968)

Fig. 11. Numerical simulations obtained with barodesy, hypoplastic model of Mašin (2005) and the MCCM; London clay data digitalised from Mašin (2005). (a) Triaxial compression tests ($p = 110 \text{ kPa} = \text{constant}$, $e_{\text{ini}} = 1.115$). (b) Isotropic compression tests

Basic characteristics such as the liquid limit w_l , the plastic limit w_p and the plasticity index I_p of the simulated clays are shown in Table 6.

Table 7 gives an overview of the numerical simulations presented in this article. Figure 11(a) shows simulations of triaxial tests with constant mean pressure starting from K_0 consolidated states. In Figs 2(b) and 11(b), simulations of isotropic compression tests are compared with experimental data. Note that the parameter κ , which governs the unloading stiffness, is used in the Mašin model and the MCCM. No corresponding parameter is used in barodesy; consequently, the unloading stiffness under isotropic compression is overestimated with barodesy (see Fig. 11(b)). Note that constitutive relations of the type $\dot{\mathbf{T}} = \mathbf{h}(\mathbf{T}, \mathbf{D}, e)$ have a simple structure and cannot capture reloading.

CONCLUSIONS

This article has shown that barodesy is able to describe important characteristics of clay behaviour for monotonic loading. Cyclic behaviour is not captured by the model presented here; this modelling is left for future publications. The constitutive equations are simple: they consist of one differential equation and one tensorial function with five constants that can be determined from well-known material parameters of soil mechanics. It was shown that barodesy comprises concepts of CSSM. Despite its mathematical simplicity, the numerical simulations are satisfactory. The

constitutive equations are easy to understand and this offers potential for further development.

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