The decay of cay?
Investigating changes in the impact of the consumption-wealth ratio on asset returns

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Over the last two decades the variable $cay$ from Lettau and Ludvigson (2001) has attracted a great amount of attention and research. Implementing the idea that the consumption-wealth ratio should have a forecasting ability for future asset returns due to a consumption smoothing behavior of rational investors, it captures deviations from a long-term trend between consumption and wealth, the latter being represented by asset wealth and labor income. We follow the traditional derivation of $cay$ (and $cday$ by Sousa (2010)), estimate them over a period ranging from 1952-2019, and use them to forecast stock market returns in excess of the “risk-free” rate. We then take a closer look at the impact of $cay$ and $cday$ in such prediction regressions over time and observe a decay in the magnitude of the coefficients. We also investigate the influence of the Covid-19 pandemic on $cay$ and its forecasting abilities as well as the relevance of a deterministic time trend in the cointegration relation between its underlying variables. Lastly, we show that out-of-sample forecasts using only $cay$ as a predictor for stock market returns would have performed worse than a prevailing mean model, casting doubt on the true forecasting power of $cay$. 
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1. Introduction

For a long time it has been a topic of empirical research to investigate the long-term predictability of asset returns and the linkage between wealth and other macroeconomic variables. This interest sparks from the fact that asset returns seem to vary with the business cycle. Various different explanations have been offered over time, from market inefficiencies (Fama (1970), Fama (1991), Fama (1998), Farmer and Lo (1999)) over agents’ responses to time-varying investment opportunities driven by variations in risk aversion (Sundaresan (1989), Constantinides (1990), Campbell and Cochrane (1999)) to the joint distribution of consumption and asset returns (Duffee (2005), Santos and Veronesi (2006)).

Another approach that gained attention over the last 20 years is the development of the economically motivated variable $cay$ by Lettau and Ludvigson (2001). It captures deviations from a long-run trend between consumption, asset wealth, and labor income. It can then be used as a proxy for the consumption-wealth ratio that should have a forecasting ability for either asset returns, consumption growth, or both, according to the underlying intertemporal consumption-based model by Campbell and Mankiw (1989). Lettau and Ludvigson (2001) attribute the forecasting power to consumption smoothing behavior by forward-looking investors and take $cay$ as a proxy that collects the conditional expectations on future asset returns by such investors.

Indeed, they show that $cay$ has a strong predictive power for aggregate stock returns in the United States. Their work inspired many others to use $cay$ and other related proxies to forecast asset returns also in other markets. For example, Fisher and Voss (2004) found evidence for the predictability of asset returns using $cay$ in Australia. Ioannidis et al. (2006) extended those findings to Canada and the United Kingdom. A larger study on 16 OECD countries by Afonso and Sousa (2011) showed that while $cay$ works well as a predictor of asset returns in many countries, it also fails in others like Austria, Germany, Italy, or Spain. They also looked at the ability of $cay$ to predict government bond returns in these 16 OECD countries, as did Sousa (2015). Caporale and Sousa (2016) also provide evidence on several emerging markets.

An important advancement of $cay$ was first introduced by Sousa (2010).
Instead of using aggregate wealth in the long-run trend relationship with consumption and labor income, Sousa (2010) used (dis)aggregate wealth, treating financial and housing wealth separately. This led to the development of cday which produces better forecasts of asset returns than its predecessor cay.

Over the last couple of years several further refinements of cay have been introduced. From a non-linear approach by Bekiros and Gupta (2015), a time-varying measure, cay_{TVP}, by Chang et al. (2019), over a Fractionally Cointegrated VAR (FCVAR) approach by Quineche (2020), to a Markov-switching version of the consumption-wealth ratio, cay_{MS}, by Bianchi et al. (2022). All of these seem to capture regime changes in the relationship between asset returns and cay better in a particular way.

The focus of this thesis, however, lies on the original measure cay from Lettau and Ludvigson (2001) and its closely related alteration cday from Sousa (2010) on U.S. data only. We will work out the underlying consumption-based theory in chapter 2 before maneuvering around some issues regarding the proxies used to capture consumption and wealth, to then estimate the long-run trend relationship between our respective variables of interest in chapter 3.

Chapter 4 provides information on the asset return data and gives a first intuition on the predictive abilities of our trend deviation measures. For example, the stock market crash in March 2020, as well as the bear market of 1973-74, were foreshadowed by a negative trend in our deviation measures. Other crashes, e.g., in 1987 or 2008, and bear markets, like in the early 2000s, do not seem to be well foretold though, see figure 2.

All this culminates in estimating forecast regressions for excess stock returns in chapter 5. After mainly following Lettau and Ludvigson (2001) and Sousa (2010) up to that part, we then investigate a phenomenon that we will frequently refer to as the “decay of cay”. It seems as though the magnitude of the predictive ability of cay and cday to forecast stock market returns has decreased at least over the last ten years if not longer. Most recently, it seems to be vanishing completely, caused by the massive disturbances in the consumption-wealth ratio due to the Covid-19 pandemic. We will not only look at the absolute impact of our measures cay and cday on future stock market returns, but also relative to rises in units of standard deviation, capturing the influence of changes in the latter over time. From this point of view, the “decay of cay” (and cday), seems to have already started shortly after the turn of the millennium and not just with the financial crisis of 2007-08.
Chapter 6 then provides robustness checks. First, we address the issue of a “look-ahead bias” in \textit{cay} introduced by the estimation of the trend relationship parameters \textit{in-sample}. This was raised by Brennan and Xia (2005) and further addressed by Hahn and Lee (2006) and Welch and Goyal (2008). In the course of this, we will provide a possible explanation for the “decay of \textit{cay}” observed in the chapter before. We follow that up with a look at the \textit{out-of-sample} performance of \textit{cay} over the last decades. Thereby, it looks as though an investor trying to use \textit{cay} as a predictor for next quarter’s stock market returns would have been better off just expecting the updated mean for the most time.

Finally, we conclude our findings in chapter 7 before providing an Appendix with additional tables and figures.

All data analyses and computations were executed with the help of the open source software program R: A Language and Environment for Statistical Computing from the R Foundation for Statistical Computing, Vienna, Austria, 2022, \url{https://www.R-project.org/}.
2. The consumption-wealth ratio

This chapter introduces the crucial linkage between consumption and wealth on the one side, and expected returns on the other side. For that, we consider a representative agent economy in which all wealth, including human capital, is tradable.

2.1. The key equation

Let $W_t$ be aggregate wealth, i.e., human capital plus asset holdings, in period $t$. Let further $C_t$ be consumption in period $t$ and $R_{w,t+1}$ the return on aggregate wealth between period $t$ and $t+1$. Then, the intertemporal consumer’s budget constraint is given by

$$W_{t+1} = R_{w,t+1} (W_t - C_t). \quad (2.1)$$

Solving forward with an infinite horizon and imposing the transversality condition that the limit of discounted future wealth is zero, Campbell and Mankiw (1989) show that

$$W_t = C_t + \sum_{i=1}^{\infty} C_{t+i} / \left( \prod_{j=1}^{i} R_{w,t+j} \right). \quad (2.2)$$

This equation just expresses that today’s wealth equals the discounted value of all future consumption.

The goal is to establish a linear relationship between log wealth, log consumption, and log returns. Therefore, from now on, let lower-case letters denote the natural logarithm of the corresponding upper-case letter variable, e.g., $w_t := \log(W_t)$, the logarithm of aggregate wealth in period $t$. Let further $\Delta$ denote the difference operator, e.g., $\Delta w_{t+1} := w_{t+1} - w_t$. Campbell and Mankiw (1989) describe the following computation, start-
ing from equation 2.1:

\[
\begin{align*}
W_{t+1} &= R_{w,t+1}(W_t - C_t) \\
\equiv \frac{W_{t+1}}{W_t} &= R_{w,t+1} \left(1 - \frac{C_t}{W_t}\right) \\
\equiv \log \left(\frac{W_{t+1}}{W_t}\right) &= \log(R_{w,t+1}) + \log \left(1 - \frac{C_t}{W_t}\right) \\
\equiv w_{t+1} - w_t &= r_{w,t+1} + \log \left(1 - \exp(c_t - w_t)\right) \quad \text{(2.3)} \\
\equiv \Delta w_{t+1} &= r_{w,t+1} + \log \left(1 - \exp(x_t)\right),
\end{align*}
\]

where \(x_t := c_t - w_t = \log(C_t/W_t)\) is the log consumption-wealth ratio. To linearize the above equation, Campbell and Mankiw (1989) take a Taylor series expansion of the function \(\log(1 - \exp(x_t))\) around the point \(x_t = x\). This leads to the approximation

\[
\log(1 - \exp(x_t)) = \log(1 - \exp(c_t - w_t)) \approx k + \left(1 - \frac{1}{\rho}\right)(c_t - w_t)
\]

where \(\rho = 1 - \exp(x) < 1\), and \(k = \log(\rho) - \left(1 - \frac{1}{\rho}\right)\log(1 - \rho)\). The parameter \(\rho\) can be interpreted as the ratio of invested wealth to total wealth, \(\frac{W - C}{W}\). Now, substituting this approximation into the above equation leads to

\[
\Delta w_{t+1} \approx k + r_{w,t+1} + \left(1 - \frac{1}{\rho}\right)(c_t - w_t),
\]

(2.3)

giving us the desired linear relationship between log wealth, log consumption and log returns. Namely, it says that the growth rate of wealth can be approximated by a constant, plus the log return on total wealth, less a small fraction, i.e., \(1 - 1/\rho\), of the log consumption-wealth ratio.

Using the fact that

\[
\Delta w_{t+1} = \Delta c_{t+1} + (c_t - w_t) - (c_{t+1} - w_{t+1})
\]

leads to

\[
c_t - w_t \approx \rho(r_{w,t+1} - \Delta c_{t+1}) + \rho(c_{t+1} - w_{t+1}) + \rho k \quad \text{(2.4)}
\]

by rearranging equation 2.3

Equation 2.4 is now further modified by solving it forward and imposing
that \( \lim_{i \to \infty} \rho^i (c_{t+i} - w_{t+i}) = 0 \) to finally obtain

\[
c_t - w_t \approx \sum_{i=1}^{\infty} \rho^i (r_{w,t+i} - \Delta c_{t+i}) + \frac{\rho k}{1 - \rho}.
\]

From here on out, we will omit unimportant linearization constants such as \( k \) and, therefore, work with the following equation:

\[
c_t - w_t = \sum_{i=1}^{\infty} \rho^i (r_{w,t+i} - \Delta c_{t+i}). \tag{2.5}
\]

Note that this equation is a log-linear version of the infinite-horizon budget constraint, cf. equation 2.2. Taking expectations conditional on information available at time \( t \) yields our key equation

\[
c_t - w_t = E_t \sum_{i=1}^{\infty} \rho^i (r_{w,t+i} - \Delta c_{t+i}). \tag{2.6}
\]

Equation 2.6 shows that changes in the consumption wealth ratio must forecast changing returns on aggregate wealth and/or changing consumption growth. In other words, the consumption-wealth ratio is a function of expected future returns on the “market portfolio”. Conversely, this means that expectations on future returns as well as changes in consumption can be inferred from observable consumption behavior. This fundamental linkage between the consumption-wealth ratio and expectations on future returns will be exploited throughout the rest of the thesis.

### 2.2. The trend deviation measures

In the previous section, we denoted the return on aggregate wealth as \( R_{w,t+1} \). However, the crucial point is to actually find a good measure for aggregate wealth, in particular, its unobservable component, human capital. To overcome this, Lettau and Ludvigson (2001) assume that the nonstationary component of human capital, denoted by \( H_t \), can be well described by aggregate labor income \( Y_t \), and, thus, implying that \( h_t = \kappa + y_t + z_t \), where \( \kappa \) is a constant and \( z_t \) is a mean zero stationary random variable. They rationalize this assumption by linking labor income to the stock of human capital in various different ways. For example, labor income can be thought of as the dividend on human capital as in Campbell (1996) and Jagannathan and Wang (1996). In this case, the return on human capital, \( R_{h,t+1} \), may be defined as \( R_{h,t+1} := (H_{t+1} + Y_{t+1})/H_t \) and a log-linear approximation of it implies
that $z_t = E_t \sum_{i=0}^{\infty} \rho_i (\Delta y_{t+i} - r_{h,t+i})$.

With this assumption, it is now possible to express the components of the consumption-aggregate wealth ratio in terms of observable variables. As mentioned in the introduction, we will consider the two similar approaches by Lettau and Ludvigson (2001) and Sousa (2010), when looking at (dis)aggregate wealth. Therefore, let $A_t$ be asset holdings, $F_t$ financial wealth, and $U_t$ housing wealth. Let further $R_{a,t}$, $R_{f,t}$, and $R_{u,t}$ be the corresponding returns, respectively. Then, we have

$$W_t = A_t + H_t = F_t + U_t + H_t,$$

and thus

$$w_t \approx \alpha_a a_t + (1 - \alpha_a) h_t,$$

(2.7)

$$w_t \approx \alpha_f f_t + \alpha_u u_t + (1 - \alpha_f - \alpha_u) h_t,$$

(2.8)

where $\alpha_a$, $\alpha_f$, and $\alpha_u$, represent the average share of asset holdings in total wealth, $A/W$, the share of financial asset holdings in total wealth, $F/W$, and the share of housing asset holdings in total wealth, $U/W$, respectively. Note that since $A_t = F_t + U_t$, we must have $\alpha_a = \alpha_f + \alpha_u$.

The return on aggregate wealth can then be decomposed into the returns of its components

$$R_{w,t} = \alpha_a R_{a,t} + (1 - \alpha_a) R_{h,t},$$

$$R_{w,t} = \alpha_f R_{f,t} + \alpha_u R_{u,t} + (1 - \alpha_f - \alpha_u) R_{h,t}.$$  

Campbell (1996) shows that the above equations can be transformed into an approximation for log returns given by

$$r_{w,t} \approx \alpha_a r_{a,t} + (1 - \alpha_a) r_{h,t},$$

(2.9)

$$r_{w,t} \approx \alpha_f r_{f,t} + \alpha_u r_{u,t} + (1 - \alpha_f - \alpha_u) r_{h,t}.$$  

(2.10)

Now, substituting equations 2.7 and 2.9 into our key equation 2.6 yields

$$c_t - \alpha_a a_t - (1 - \alpha_a) h_t = E_t \sum_{i=1}^{\infty} \rho^i (\alpha_a r_{a,t+i} + (1 - \alpha_u) r_{h,t+i} - \Delta c_{t+i}).$$

Clearly, this equation still contains the unobservable variable $h_t$. Using the connection to log labor income established above, it can be replaced
by $\kappa + y_t + z_t$, yielding

$$c_t - \alpha_a a_t - (1 - \alpha_a) y_t = E_t \sum_{i=1}^{\infty} \rho^i \left( [\alpha_a r_{a,t+i} + (1 - \alpha_a) r_{h,t+i}] - \Delta c_t \right)$$

$$+ (1 - \alpha_a) z_t,$$

again omitting the unimportant constant $\kappa$. Since all the terms on the right-hand side of this equation are assumed to be stationary, $c$, $a$, and $y$ must be cointegrated. Thus, the left-hand side gives the deviation in the common trend of $c_t$, $a_t$, and $y_t$. Following [Lettau and Ludvigson (2001)] , this trend deviation term will from here on forward be denoted by

$$cay_t := c_t - \alpha_a a_t - (1 - \alpha_a) y_t. \quad (2.11)$$

Completely analogously to the construction of $cay_t$, [Sousa (2010)] defines

$$cday_t := c_t - \alpha_f f_t - \alpha_u u_t - (1 - \alpha_f - \alpha_u) y_t. \quad (2.12)$$

As long as expected future returns on human capital, $r_{h,t+i}$, and consumption growth, $\Delta c_t$, are not too variable, or at least highly correlated with expected returns on assets (both financial and housing in case of $cday_t$), $cay_t$ will be a good proxy for market expectations on future asset returns, $r_{a,t+i}$, while $cday_t$ will be a good proxy for market expectations on future financial, $r_{f,t+i}$, and housing asset returns, $r_{u,t+i}$.

The goal of the next chapter will be to create consistent estimates for $\alpha_a$, $\alpha_f$, and $\alpha_u$ to accurately capture the long term trend among consumption, asset holdings, and labor income.
3. The estimation of the trend relationship

Following Lettau and Ludvigson (2001), we exploit the asymptotic properties of cointegrated variables in order to find consistent estimates for the shared trend in consumption, asset holdings, and labor income. Before we can get to that, we have to deal with a measurement issue arising when trying to capture the flow of consumption. This will be dealt with in the following two sections.

3.1. The measurement issues around consumption

In their original paper, Lettau and Ludvigson (2001) argue, referring to previous empirical work on consumption-based models, that using expenditures on nondurables and services (NDS) fits best to the theoretical framework of the flow of consumption. From this point of view, expenditures on durable goods are rather seen as replacements and additions to existing stock and, therefore, should not be taken into account when measuring consumption. But, nondurables and services represent only one component of total consumption flow, with the other one being the service flow from the stock of household durable goods, which is unobservable. To cope with this, Lettau and Ludvigson (2001) follow Blinder and Deaton (1985) and Galí (1990) in assuming that (log) total consumption is a constant multiple of (log) nondurables and services consumption. However, in an updated note, Lettau and Ludvigson (2019) present evidence that the assumption of NDS being a stable fraction of total consumption is no longer valid, as the ratio of NDS over total personal consumption expenditures (PCE) substantially decreased over the last 40 years. They argue that the NDS proxy omits an increasingly important part of the consumption flow which is funded from assets $a_t$ and labor income $y_t$ and, thus, $cay_t$ cannot be expected to be a valid cointegrating residual anymore. They even provide a theory for the divergence between NDS and PCE expenditures in saying that the sharply rising income and wealth unequal-
ities over that period led to an increase in the consumption of luxury goods which are disproportionally durable goods. In the absence of any satisfactory proxies for the service flow from the household durable stock, Lettau and Ludvigson (2019) settle with total personal consumption expenditures (PCE) as their measure of $c_t$ in $cay_t$. They also provide conditions under which this is justified which we will present in the following section.

3.2. Cointegration with total personal consumption expenditures

This section shows how using total personal consumption expenditures instead of the true, but unobservable, total consumption flow leads to a valid cointegration relation with $a_t$ and $y_t$. Building up on this groundwork, we can then estimate the long term trend among our variables in the next section.

Starting off at the equation for the evolvement of the household durables capital stock

$$K_{t+1} = (1 - \delta_t)K_t + I_t$$

where $I_t$ stands for investment, measured as expenditures on household durable goods, and $\delta_t$ for the rate of depreciation. Lettau and Ludvigson (2019) assume that the unobserved durables service flow $C^k_t$ is related to the capital stock according to

$$C^k_t = (\delta_t + q_t)K_t$$

(3.1)

where $q_t$ is a latent component of the service flow that is not driven by depreciation. They further assume that both $\delta_t$ and $q_t$ are stationary random variables. This gives

$$C^k_t = \delta_t K_t + q_t K_t$$

$$\Leftrightarrow C^k_t = K_t - K_{t+1} + I_t + q_t K_t$$

$$\Leftrightarrow C^k_t = (1 + q_t)K_t - K_{t+1} + I_t$$

$$\Leftrightarrow C^k_t = \chi_t + I_t$$

where $\chi_t := (1 + q_t)K_t - K_{t+1}$.

Total personal consumption expenditures are defined as

$$C_t^{\text{PCE}} := C_t^{\text{NDS}} + I_t$$
where $C_t^{NDS}$ is expenditures on nondurables and services. This leads to total consumption, $C_t$, from the intertemporal budget constraint, cf. equation 2.1 being

\[ C_t = C_t^k + C_t^{NDS} = \chi_t + I_t + C_t^{NDS} = \chi_t + C_{t}^{PCE}. \]

Lettau and Ludvigson (2019) then consider the special case of $C_t^k = \delta_t K_t$, $\chi_t = -(K_{t+1} - K_t)$, and assume that capital $K_t$ is stationary in log differences, or that the ratio $K_{t+1}/K_t = \exp(\epsilon_t)$ is stationary. Then, the difference in levels, $K_{t+1} - K_t = \exp(\epsilon_t)K_t - K_t = K_t(\exp(\epsilon_t) - 1)$ is not stationary. Instead, it grows with the unit root in $K_t$. They claim that the same logic holds for the more general case in equation 3.1. Therefore, they assume that $\chi_t$ is nonstationary. Converting to logs leads to

\[
\log(C_t) = \log \left( C_{t}^{PCE} + \chi_t \right) = \log \left[ C_{t}^{PCE} \left( 1 + \frac{\chi_t}{C_{t}^{PCE}} \right) \right] = \log \left( C_{t}^{PCE} \right) + \log \left( 1 + \frac{\chi_t}{C_{t}^{PCE}} \right).
\]

Under the assumption that $\chi_t/C_{t}^{PCE}$ is stationary, a first-order Taylor approximation gives

\[
\log \left( 1 + \frac{\chi_t}{C_{t}^{PCE}} \right) \approx k + \frac{\chi_t/C_{t}^{PCE}}{1 + \chi_t/C_{t}^{PCE}} \left( \log(\chi_t) - \log \left( C_{t}^{PCE} \right) \right).
\]

Again omitting the unimportant constant $k$, Lettau and Ludvigson (2019) get

\[
\log(C_t) = \log \left( C_{t}^{PCE} \right) + \frac{\chi_t/C_{t}^{PCE}}{1 + \chi_t/C_{t}^{PCE}} \left( \log(\chi_t) - \log \left( C_{t}^{PCE} \right) \right). \tag{3.2}
\]

Now, if $\chi_t/C_{t}^{PCE}$ is stationary, then log of total consumption is cointegrated with log of total PCE since the remaining term on the right-hand side in equation 3.2 is stationary. Hence, one can use log $\left( C_{t}^{PCE} \right)$ instead of log $C_t$. The rest of log total consumption is then subsumed into the stationary error term $e_{ay_t}$.

Lettau and Ludvigson (2019) stress that the assumptions given above lead to two testable restrictions. First, if $\log(\chi_t)$ and log $\left( C_{t}^{PCE} \right)$ are cointegrated with any vector other than $(1, -1)'$, log $C_t$ and log $\left( C_{t}^{PCE} \right)$
will not be cointegrated when looking at equation 3.2 due to the fact that the right-hand term will not be stationary. Thus, using log \( C_{t}^{\text{PCE}} \) as a measure of \( c_t \) in \( cay_t \) implies that \( cay_t \) will not be stationary. To put it another way, the assumptions imply that \( cay_t \) must be stationary when using log \( C_{t}^{\text{PCE}} \) as a consumption measure. Second, the cointegrating coefficients for \( a_t \) and \( y_t \) should sum up to one when using \( C_{t}^{\text{PCE}} \) as the measure for consumption. We will revisit these testable restrictions in the following section to check whether the above assumptions seem plausible.

### 3.3. The estimation of the long-run trend relationship

After establishing a reasonable measure for consumption under certain assumptions, namely total personal consumption expenditures, we can now turn our attention towards estimating the aforementioned trend relationship among consumption, asset wealth, and labor income. The data used in the following are quarterly per capita variables, measured in 2012 dollars. Appendix A.1 will provide more details on the sources of the data as well as the precise construction of the variables in use.

The first step, as in Lettau and Ludvigson (2001), is to test whether our time series contain a unit root. This is done by applying Augmented Dickey-Fuller (ADF) tests. It seems that all variables contain a unit root. At the same time, when applying an ADF test to the differences of the variables, they appear to be stationary. Hence, we conclude that consumption, household net worth, financial net worth, housing net worth, and labor income are \( I(1) \), i.e., first-order integrated. Note again that we always refer to the log of these variables after the transformation executed in section 2.1.

The next step is to test for cointegration among the variables using the procedure suggested by Johansen (1991). The evidence points into the direction of a single cointegrating relationship between our variables and is presented in detail in appendix A.2. By that, it also supports the use of personal consumption expenditures as our measure for consumption. In order to cope with the problem of regressor endogeneity affecting the distribution of the least squares estimator, we follow Lettau and Ludvigson (2001) in employing a dynamic least squares method (DLS) that adds leads and lags of the first differences in our variables to a standard ordinary least squares (OLS) regression of consumption on asset holdings (financial and housing assets, respectively) and labor income. Thus, we
estimate the following equations using standard OLS:

\[ c_t = \mu + \beta_a a_t + \beta_y y_t + \sum_{i=-k}^{k} b_{a,i} \Delta a_{t-i} + \sum_{i=-k}^{k} b_{y,i} \Delta y_{t-i} + \epsilon_t, \quad (3.3) \]

for aggregate wealth as in Lettau and Ludvigson (2001), and

\[ c_t = \mu + \beta_f f_t + \beta_u u_t + \beta_y y_t \\
+ \sum_{i=-k}^{k} b_{f,i} \Delta f_{t-i} + \sum_{i=-k}^{k} b_{u,i} \Delta u_{t-i} + \sum_{i=-k}^{k} b_{y,i} \Delta y_{t-i} + \epsilon_t, \quad (3.4) \]

for disaggregate wealth as in Sousa (2010).

The exact number of leads and lags to be used differs within the literature and will also do so within this thesis. It is important to note that the main results are not significantly affected by that, but we will nevertheless always report \( k \), whenever regression results are presented.

An important feature of the above specification, pointed out by Lettau and Ludvigson (2001), is that even though \( \epsilon_t \) will typically be correlated with the regressors \( a_t \) and \( y_t \) \( (f_t, u_t \) and \( y_t \), respectively), the estimates \( \beta_a \) and \( \beta_y \) \( (\beta_f, \beta_u, \) and \( \beta_y, \) respectively) will still be consistent due to a faster convergence rate thanks to the cointegrating relationship between the variables, cf. Stock (1987). Thus, the \( \beta \)'s should provide consistent estimates of the \( \alpha \)'s from equations 2.11 and 2.12, namely the average ratios of each component of wealth.

Finally, implementing the regressions in 3.3 and 3.4 using data from the first quarter of 1952 to the fourth quarter of 2019, we end up with the following parameters of the shared trend (ignoring coefficient estimates of the first differences):

\[ c_t = -0.434 + 0.210a_t + 0.810y_t, \quad (3.5) \]
\[ c_t = -0.264 + 0.103f_t + 0.096u_t + 0.828y_t. \quad (3.6) \]

Here, a model selection based on information criteria led to \( k = 1 \) being used. Besides, note that we followed Sousa (2010) in shifting the observations of time series that contain end-period values to the beginning of the subsequent period.

From here on out, we denote the estimated trend deviation as

\[ \hat{cay}_t := c_t - \hat{\beta}_a a_t - \hat{\beta}_y y_t, \]
\[ \hat{cday}_t := c_t - \hat{\beta}_f f_t - \hat{\beta}_u u_t - \hat{\beta}_y y_t \]
where $\beta_a, \beta_y, \beta_f$ and $\beta_u$ are the estimated parameters from equations 3.3 and 3.4, respectively.

Jumping back to the second of the testable restrictions in the previous section that follow from the assumptions under which we can use total personal consumption expenditures (PCE) as a proxy for total consumption, we now clearly see that the sum of our estimates is pretty close to one in both cases, meaning the data support the assumptions made. For example, the average share of income in total wealth, $1 - \alpha_a$, is about 81% when looking at aggregate wealth.

Another interesting immediate takeaway is that when comparing our regression results with the ones from Sousa (2010), we have a much smaller, in fact almost negligible, difference in the long-run elasticities of consumption with financial and housing wealth, 0.103 and 0.096 versus 0.16 and 0.02. After running a couple of reference regressions, this seems to be rather a feature of changes over the longer time horizon we consider than of the change in methodology as described in the previous two sections. Whether that points towards a diminishing effect in using (dis)aggregate wealth over aggregate wealth remains to be seen in what follows.

### 3.4. The short-term dynamics of the estimation

In the previous section, we have specified and estimated our long-run regression equations according to Lettau and Ludvigson (2001) and Sousa (2010). Our focus now shifts towards the short-term dynamics and how deviations from the long-term trend can be interpreted. We will pursue the question whether such deviations are better described as transitory movements in asset wealth or as transitory movements in consumption and labor income. To answer this, we follow both Lettau and Ludvigson (2001) and Sousa (2010) in estimating a three-variable cointegrated vector autoregression (VAR), four-variable, respectively, in which the log differences of our variables are each regressed on the one-period lagged log differences of all variables and an “error-correction term”, being the estimated lagged trend deviation $\hat{c}_{ay_{t-1}}$, $\hat{c}_{dy_{t-1}}$, respectively. The results are reported in tables 1 and 2.

Our main focus lies on the coefficients of the trend deviation measures and the corresponding significance levels. When looking at table 1, it appears that consumption growth is mostly driven by past changes in asset holdings and income rather than past consumption growth. This result significantly differs from what Lettau and Ludvigson (2001) found back in 2001. However, the coefficient of $\hat{c}_{ay_{t-1}}$ is still negative and, more
importantly, not significant at the 10% level. If we now recall our key equation 2.6, it appears that deviations from the long-run equilibrium between consumption, asset holdings, and income do not seem to affect consumption growth. It is more so the case that only asset growth is significantly affected by deviations from the long-run equilibrium. This means deviations from the shared trend between consumption, asset holdings, and labor income can rather be interpreted as transitory movements in asset wealth than as transitory movements in consumption or labor income. In the following chapters, we will see that this follows from the fact that the trend deviation measure $\hat{c}_t - \hat{a}_t$ forecasts asset returns, very much in line with the theory presented above, cf. equation 2.6. In the regression of income changes, see the third column in table 1, it is still past consumption growth that is the main driving factor, albeit weaker than in Lettau and Ludvigson (2001).

<table>
<thead>
<tr>
<th></th>
<th>$\Delta c_t$</th>
<th>$\Delta a_t$</th>
<th>$\Delta y_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta c_{t-1}$</td>
<td>0.140*</td>
<td>0.364*</td>
<td>0.396***</td>
</tr>
<tr>
<td></td>
<td>(1.775)</td>
<td>(1.958)</td>
<td>(3.969)</td>
</tr>
<tr>
<td>$\Delta a_{t-1}$</td>
<td>0.076***</td>
<td>0.116</td>
<td>-0.016</td>
</tr>
<tr>
<td></td>
<td>(2.633)</td>
<td>(1.411)</td>
<td>(-0.429)</td>
</tr>
<tr>
<td>$\Delta y_{t-1}$</td>
<td>0.185***</td>
<td>0.039</td>
<td>-0.113</td>
</tr>
<tr>
<td></td>
<td>(3.248)</td>
<td>(0.244)</td>
<td>(-1.088)</td>
</tr>
<tr>
<td>$\hat{c}_t \hat{a}_t$</td>
<td>-0.008</td>
<td>0.100*</td>
<td>0.018</td>
</tr>
<tr>
<td></td>
<td>(-0.392)</td>
<td>(1.775)</td>
<td>(0.712)</td>
</tr>
<tr>
<td>(Intercept)</td>
<td>0.000</td>
<td>0.047*</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>(-0.047)</td>
<td>(1.926)</td>
<td>(1.059)</td>
</tr>
</tbody>
</table>

| Adjusted $R^2$ | 0.161 | 0.058 | 0.064 |

Note: *p<0.1; **p<0.05; ***p<0.01


A somewhat different picture evolves when looking at table 2. Here, $\hat{c}_t \hat{a}_t$ is not only significant in the financial wealth regression, but also in the income regression. However, the coefficient in the latter is
much smaller and also only significant at the 10% level. Thus, the main takeaways remain the same - deviations in financial wealth from the long-term trend with consumption, housing wealth, and labor income uncover an important transitory variation, and, at the same time, the trend deviation does not predict consumption growth. Similar to what Sousa (2010) found, the housing wealth regression is the strongest in terms of adjusted $R^2$, and changes in housing wealth are mainly driven by past changes of its own. As Sousa (2010) mentions, this is in line with past work by Case and Shiller (1989) and Ortalo-Magne and Rady (2006), showing that housing returns are highly autocorrelated.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta c_t$</th>
<th>$\Delta f_t$</th>
<th>$\Delta u_t$</th>
<th>$\Delta y_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta c_{t-1}$</td>
<td>0.125</td>
<td>0.315</td>
<td>0.375</td>
<td>0.365***</td>
</tr>
<tr>
<td></td>
<td>(1.590)</td>
<td>(1.110)</td>
<td>(1.642)</td>
<td>(3.800)</td>
</tr>
<tr>
<td>$\Delta f_{t-1}$</td>
<td>0.038**</td>
<td>0.038</td>
<td>0.026</td>
<td>-0.027</td>
</tr>
<tr>
<td></td>
<td>(2.391)</td>
<td>(0.483)</td>
<td>(0.665)</td>
<td>(-1.264)</td>
</tr>
<tr>
<td>$\Delta u_{t-1}$</td>
<td>0.045**</td>
<td>0.132</td>
<td>0.562***</td>
<td>0.044</td>
</tr>
<tr>
<td></td>
<td>(2.364)</td>
<td>(1.579)</td>
<td>(6.569)</td>
<td>(1.438)</td>
</tr>
<tr>
<td>$\Delta y_{t-1}$</td>
<td>0.179***</td>
<td>0.003</td>
<td>-0.348*</td>
<td>-0.116</td>
</tr>
<tr>
<td></td>
<td>(3.068)</td>
<td>(0.013)</td>
<td>(-1.922)</td>
<td>(-1.131)</td>
</tr>
<tr>
<td>$\hat{c}_{dayt-1}$</td>
<td>-0.006</td>
<td>0.184*</td>
<td>0.012</td>
<td>0.044*</td>
</tr>
<tr>
<td></td>
<td>(-0.260)</td>
<td>(1.965)</td>
<td>(0.200)</td>
<td>(1.715)</td>
</tr>
<tr>
<td>(Intercept)</td>
<td>0.002</td>
<td>0.052**</td>
<td>0.006</td>
<td>0.015**</td>
</tr>
<tr>
<td></td>
<td>(0.306)</td>
<td>(2.101)</td>
<td>(0.343)</td>
<td>(2.230)</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.170</td>
<td>0.030</td>
<td>0.333</td>
<td>0.080</td>
</tr>
</tbody>
</table>


We specifically want to point out that both tables present a much smaller coefficient estimate for the error-correction term in the asset growth regression (financial asset growth, respectively) than the results from Lettau and Ludvigson (2001) and Sousa (2010) did, namely 0.1 vs. 0.445 and 0.184 vs. 0.304, respectively. This is in line with, and further points
towards, the “decay of cay” we will be addressing in chapter 5. Even though the effects seem to have weakened over time, they are still existent and support the theoretical framework discussed above. Besides that, splitting up asset wealth into its two components, financial and housing wealth, appears to strengthen the effects of our trend deviation measure as the coefficient in the financial wealth regression (0.184) is higher than the one in the aggregate wealth regression (0.1). It appears that deviations from the shared trend are best described as transitory movements in financial wealth and not housing wealth. This is further underlined by looking at figure 1 which displays the log per capita series of our variables over time. Clearly, the housing wealth time series, $u_t$, seems to behave on its own a bit, which is why Sousa (2010) used (dis)aggregate wealth in the first place.

Figure 1.: Time series of log per capita consumption, asset wealth, financial wealth, housing wealth, and labor income. Series comprise the period 1952:1-2019:4.
4. The asset return data

In this chapter, we want to briefly discuss the asset data in use and give short summary statistics as well as a graphical illustration of our trend deviation measures in connection with asset returns. The theoretical framework discussed in chapter 2, cf. key equation 2.6, deals with returns to the “market portfolio”. Now, as there is no real life measure for the whole U.S. stock market, we settle with using historical data on one of the broader indices available. All regression results presented within this thesis use the Center for Research in Security Press (CRSP) Value-Weighted Index for the S&P 500 Universe. To capture all returns, including dividends, we use the Total Return Index provided by the CRSP, which assumes a maximal reinvestment of all distributions. The original series contains daily index levels from December 31st, 1925, to the 31st of March, 2022. Quarterly returns are then computed using only the last trading days of each quarter, leading to a total of 385 observations. However, since our time series on consumption, asset wealth, and labor income are only available from the first quarter of 1952 onwards, we will only be using the asset data from there on, too. In chapter 5, we will discuss the impact of the Covid-19 pandemic in detail and why we are only using the data on consumption, asset wealth, and labor income until the fourth quarter of 2019 because of it. Nevertheless, we still include the first quarter of 2020 in our asset data as this makes sense given that we are interested in the forecasting abilities of $\hat{c}_t$ and $\hat{d}_t$, and to have one more observation available.

In line with the notation in chapter 2, $r_t$ will denote the log real return between period $t - 1$ and $t$. In order to convert the nominal returns computed from the index time series to the real returns we are interested in, we use the total personal consumption expenditures price deflator from the Bureau of Economic Analysis (BEA) as our inflation measure. Similar to Lettau and Ludvigson (2001) and Sousa (2010), we are more interested in predicting excess stock returns rather than real returns, even though the overall results are quite similar. We follow Sousa (2010) in defining $r_{f,t}$ as the log real yield rate of the 3-month treasury bill (the “risk-free” rate). Then, the log excess return is given by $r_t - r_{f,t}$. More information on the “risk-free” rate and the price deflator is given in appendix A.1.
Next, we want to give brief summary statistics of our data as well as a graphical illustration of them. The sample contains 272 observations for our trend deviation measures and one more for the excess returns. In table 3., we see that the mean log excess return is about 0.0146 which is roughly equivalent to 1.5% per quarter or 6% annually. The standard deviation is fairly high with around 8 percentage points quarterly, while the autocorrelation has become even lower than the 0.12 for the sample of 1952:4-1998:3 that Lettau and Ludvigson (2001) used.

On the other hand, the standard deviation and autocorrelation of \( \hat{cay}_t \) are both significantly higher than what Lettau and Ludvigson (2001) found with around 0.011 and 0.79, respectively. However, this may also be due to the changes in methodology in the construction of the deviation measures described in chapter 3. The same holds for the values of \( \hat{cday}_t \).

Table 3.: Univariate summary statistics. The sample period is 1952:1-2019:4 for \( \hat{cay}_t \) and \( \hat{cday}_t \), 1952-2020 for \( r_t - r_{f,t} \), respectively.

<table>
<thead>
<tr>
<th></th>
<th>( r_t - r_{f,t} )</th>
<th>( \hat{cay}_t )</th>
<th>( \hat{cday}_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0146</td>
<td>-0.4349</td>
<td>-0.2650</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.0796</td>
<td>0.0193</td>
<td>0.0186</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.074</td>
<td>0.917</td>
<td>0.904</td>
</tr>
</tbody>
</table>

Table 4. provides the correlations between the three time series. We see that the correlation of excess returns with \( \hat{cay}_t \) and with \( \hat{cday}_t \) are fairly similar and weak, while the two measures themselves are (unsurprisingly) highly correlated.

Table 4.: Correlation matrix. The sample period is 1952:1-2019:4.

<table>
<thead>
<tr>
<th></th>
<th>( r_t - r_{f,t} )</th>
<th>( \hat{cay}_t )</th>
<th>( \hat{cday}_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_t - r_{f,t} )</td>
<td>1.000</td>
<td>0.119</td>
<td>0.123</td>
</tr>
<tr>
<td>( \hat{cay}_t )</td>
<td>1.000</td>
<td>0.848</td>
<td></td>
</tr>
<tr>
<td>( \hat{cday}_t )</td>
<td></td>
<td></td>
<td>1.000</td>
</tr>
</tbody>
</table>

To emphasize the predictive ability of our deviation measures even further, we again follow Lettau and Ludvigson (2001) in plotting the standardized trend deviation measures together with the standardized excess returns in figure 2. Note that we included one more observation from the first Covid-19 impacted quarter of 2020 in the excess return time series as we want to uncover any forecasting abilities of our trend deviation measures. It illustrates that often positive deviations preceded large positive excess returns while negative ones often preceded large negative returns.
Figure 2.: Standardized trend deviation measures and excess returns over the sample period of 1952:1-2019:4, including the one observation on excess returns for the first Covid-19 impacted quarter of 2020.

More precisely, especially in the 1950s and 1960s, spikes in excess returns seem to be preceded by an according turn in our deviation measures. The two periods in which both $\hat{c}_\text{ay}_t$ and $\hat{c}_\text{day}_t$ display a negative trend over a longer time horizon, namely from the mid-60s to mid-70s and from the early 2010s to 2020, were followed shortly after by large negative returns in 1974:3 and 2020:1. On the other hand, the stock market crashes of 1987 and during the financial crisis in 2008 were not foreshadowed by negative swings in our deviation measures - neither was the bear market of the early 2000s after the burst of the “dotcom bubble”, even though it seems that in this particular case $\hat{c}_\text{day}_t$ was a better harbinger than $\hat{c}_\text{ay}_t$ and both at least showed a small decline prior to and during the bear market.

All in all, both measures behaved fairly similar throughout time with slight differences during certain periods. The biggest difference occurred right after the financial crisis when $\hat{c}_\text{day}_t$ rose to its all-time high while $\hat{c}_\text{ay}_t$ exhibited a slow decline. This can be explained by the slump in housing wealth, $u_t$, depicted by the orange line in figure[1] after the burst of the housing bubble in the United States, together with the fact that this is stronger accounted for in $\hat{c}_\text{day}_t$ than in $\hat{c}_\text{ay}_t$ using aggregate wealth.
5. Forecasting regressions

5.1. H-period forecasts

In the previous chapters, we established two proxies for the deviation from the long term trend in the consumption-wealth ratio and further defined how we measure returns on a broad stock market index in excess of the “risk-free” rate. Our goal in this chapter is to assess the ability of our proxies to forecast excess returns according to the theory laid out in chapter 2, culminating in equation 2.6. In order to do that, we again follow Lettau and Ludvigson (2001) in defining the $H$-period log excess return as $r_{t+1} - r_{f,t+1} + \cdots + r_{t+H} - r_{f,t+H}$, i.e., the sum of excess returns over the next $H$ periods. This will then always be regressed on the current level of our proxies $\hat{cay}_t$ and $\hat{cday}_t$. Note that for $H = 1$ we get quarterly ahead predictions as separately presented in Lettau and Ludvigson (2001). We also ran regressions of excess returns on their one-period lagged values solely, as well as ones that added those lags to a quarterly-ahead prediction using $\hat{cay}_t$ and $\hat{cday}_t$, respectively. The estimates for the lagged excess returns are not significant in either case which is why we followed Lettau and Ludvigson (2001) and Sousa (2010) in only considering the regressions on our proxies and a constant. Table 5 displays the results of the forecasting regressions using $\hat{cay}_t$ for a horizon of up to one year, i.e., $H = 4$. The equivalent results for $\hat{cday}_t$ are presented in table 6. What strikes the eye the most are of course the coefficient estimates for the quarterly-ahead predictions of 0.579 and 0.663, respectively. These are significantly lower than what Lettau and Ludvigson (2001) found for $\hat{cay}_t$ with 2.16 and Sousa (2010) for $\hat{cday}_t$ with 1.018. It must be noted though that both papers of course used much shorter sample periods as well as the outdated consumption measure instead of total personal consumption expenditures. A direct comparison is, therefore, a bit delicate and has to be handled carefully. To circumnavigate this issue, we will compute our own estimates for different sample periods in the following sections. The comparison with an estimate of 0.74 in the updated results from Lettau and Ludvigson (2019), using data from 1952:2-2014:3 and the same consumption measure as we do, looks much more encouraging.
Dependent variable: \( r_{t+1} - r_{f,t+1} + \cdots + r_{t+H} - r_{f,t+H} \)

<table>
<thead>
<tr>
<th>( H = 1 )</th>
<th>( H = 2 )</th>
<th>( H = 3 )</th>
<th>( H = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{cay}_t )</td>
<td>0.579**</td>
<td>0.981*</td>
<td>1.466**</td>
</tr>
<tr>
<td>(2.220)</td>
<td>(1.825)</td>
<td>(2.145)</td>
<td>(2.405)</td>
</tr>
<tr>
<td>(Intercept)</td>
<td>0.266**</td>
<td>0.456*</td>
<td>0.682**</td>
</tr>
<tr>
<td>(2.358)</td>
<td>(1.946)</td>
<td>(2.265)</td>
<td>(2.536)</td>
</tr>
</tbody>
</table>

Adjusted \( R^2 \) 0.016 0.023 0.035 0.043

Note: *p<0.1; **p<0.05; ***p<0.01

Table 5.: Forecasting regression estimates for \( \hat{cay}_t \) spanning horizons from one quarter up to one year. Newey and West (1987) corrected t-statistics appear in parenthesis. The sample period is 1952:1-2019:4 for \( \hat{cay}_t \), 1952:1-2020:1 for excess returns, respectively.

It is noteworthy that the adjusted \( R^2 \) is also much smaller than in the benchmark papers. We get an explained variation of only 1.6% and 2%, respectively, whereas Lettau and Ludvigson (2019) and Sousa (2010) had 3%.

Dependent variable: \( r_{t+1} - r_{f,t+1} + \cdots + r_{t+H} - r_{f,t+H} \)

<table>
<thead>
<tr>
<th>( H = 1 )</th>
<th>( H = 2 )</th>
<th>( H = 3 )</th>
<th>( H = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{cday}_t )</td>
<td>0.663**</td>
<td>1.152**</td>
<td>1.741**</td>
</tr>
<tr>
<td>(2.338)</td>
<td>(2.057)</td>
<td>(2.334)</td>
<td>(2.690)</td>
</tr>
<tr>
<td>(Intercept)</td>
<td>0.190**</td>
<td>0.335**</td>
<td>0.506**</td>
</tr>
<tr>
<td>(2.534)</td>
<td>(2.249)</td>
<td>(2.528)</td>
<td>(2.906)</td>
</tr>
</tbody>
</table>

Adjusted \( R^2 \) 0.020 0.030 0.046 0.059

Note: *p<0.1; **p<0.05; ***p<0.01

Table 6.: Forecasting regression estimates for \( \hat{cday}_t \) spanning horizons from one quarter up to one year. Newey and West (1987) corrected t-statistics appear in parenthesis. The sample period is 1952:1-2019:4 for \( \hat{cday}_t \), 1952:1-2020:1 for excess returns, respectively.
A similar picture evolves when looking at a one year horizon. Here, we get estimates of 1.869 and 2.275, respectively. Lettau and Ludvigson (2001), Sousa (2010), and Lettau and Ludvigson (2019) had estimates of 6.72, 3.919, and 3.23, respectively. Also, our adjusted $R^2$ of 4.3% and 5.9%, respectively, is lower than the 9-18% span in the aforementioned papers. Again, the same obstacles as described above apply here, but nonetheless we already get a glimpse into the “decay of $\hat{c}ay$” over time. Besides all that, we can confirm the findings of Sousa (2010) that in general $\hat{c}ay_t$ outperforms $\hat{c}ay_t$ by a notable margin, both in terms of stronger estimates, including t-statistics, and adjusted $R^2$. This, however, seems to be largely dependent on the number of leads/lags, $k$, we use in the estimation of the long-run trend relationship. Table 7 shows the influence of $k$ on the coefficient estimate for one-quarter ahead predictions using $\hat{c}day_t$, while it also highlights that the coefficient estimate for $\hat{c}ay_t$ is seemingly almost unaffected in comparison.

<table>
<thead>
<tr>
<th>$k$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{c}ay_t$</td>
<td>0.579</td>
<td>0.580</td>
<td>0.582</td>
<td>0.584</td>
<td>0.585</td>
<td>0.586</td>
<td>0.586</td>
<td>0.587</td>
</tr>
<tr>
<td>$\hat{c}day_t$</td>
<td>0.663</td>
<td>0.664</td>
<td>0.660</td>
<td>0.654</td>
<td>0.641</td>
<td>0.624</td>
<td>0.598</td>
<td>0.566</td>
</tr>
</tbody>
</table>

Table 7.: Coefficient estimates for $\hat{c}ay_t$ and $\hat{c}day_t$ in one-quarter ahead forecast regressions of excess returns. $k$ represents the number of leads/lags used in the estimation of the long-run trend relationship between the respective variables. The sample period is 1952:1-2019:4 for the deviation measures and 1952:1-2020:1 for excess returns.

To make the interpretation of the above estimates from tables 5 and 6 a bit more intuitive, we recall from table 3 that the standard deviation of both our trend deviation measures is roughly 0.02. This means, for example, that a one standard deviation increase in $\hat{c}ay_t$ leads to an estimated rise in excess returns one quarter ahead of roughly 116 basis points. This amounts to an increase of around 4.6 percentage points annually. For $\hat{c}day_t$ the same effect is roughly 5.3 percentage points annually. Compared to what Lettau and Ludvigson (2001) found with roughly 9 percentage points annually, this is again lower. Appendix A.3 will provide further estimates for longer forecast horizons of up to 5 years in table A.3.

One last thing we have to keep in mind is that the theory allows for another interpretation of deviations from the long-run trend among our variables, namely that it forecasts consumption growth rather than asset returns or at least both, cf. equation 2.6. To address this issue, we also ran regressions of the $H$-period consumption growth $\Delta c_{t+1} + \cdots + \Delta c_{t+H}$ on our trend deviation measures $\hat{c}ay_t$ and $\hat{c}day_t$ for various time horizons up to $H = 16$. Neither the coefficient estimates for $\hat{c}ay_t$ nor the ones for
were significant even at the 10\% level for any of the time horizons. Thus, we can conclude with a high degree of certainty that our deviation measures actually forecast only asset returns and not consumption growth. The exact results can be found in table A.4.

5.2. The decay of cay

We have mentioned the “decay of cay” multiple times throughout the thesis already, and with the work done so far we are now in the position to finally address this issue properly. As mentioned in the previous section, it seems as though the coefficient estimates of \( \hat{cay}_t \) and \( \hat{cday}_t \) in forecasting regressions of excess returns declined over time. The goal of this section will be to properly confirm this intuition by using the same methodology in the construction of our deviation measures for different time horizons. More precisely, we will run forecast regressions of excess returns on both our deviation measures just like in the previous section, but we will add one quarter of observations step by step. This way, we simulate the possible level of knowledge on all relevant data up to that point in time. Note that this may seem as a proper out-of-sample (OOS) forecast at first glance. However, we will always be using the full sample available up to a certain point to estimate our deviation measures which, of course, could not have been completely known earlier in time. Thus, we do adopt a somewhat in-between approach, mixing in-sample forecasts per se with information available up to a certain point in time only. This is often described as a “look-ahead bias”.

To get a first idea of what the end results of this will look like, figure 3 shows the point estimates of \( \hat{cay}_t \) for quarterly ahead predictions over time. One problem with this technique is that it requires a large number of data to ensure consistency of the estimates, cf. \textit{Lettau and Ludvigson (2001)}. This explains the massive variation in the early parts of figure 3. We decided to go back until the first quarter of 1970 for our time-varying deviation measures. To stay consistent with our estimation of the long-run trend between consumption, asset wealth, and labor income, we will again use leads and lags of up to \( k = 1 \). Other than in the previous section, we do include the Covid-19 impacted quarters now as well, with our last observation being the first quarter of 2022. The reasons for excluding this so far and being cautious with it when looking at the “decay of cay” are explained and shown in more detail in section 5.4.
Figure 3.: Coefficient estimates for $\hat{c}_{\Delta y_i}$ in the quarterly-ahead predictions of excess returns, re-estimated each quarter from 1970:1-2022:1. The sample starts in 1952:1.

Clearly, in the early periods there is a large variation due to the small number of observations being used for the estimation of $\hat{c}_{\Delta y_i}$. However, the estimate seems to stay in a corridor around one from the late 80s until around 2010. From there on out, we observe a linear decay in the coefficient estimate, spiking one last time with an estimate of 0.576 in the first quarter of 2020, before falling into irrelevance thereafter. Note that this estimate is slightly weaker than the 0.579 we have observed in table 5 because here the first Covid-19 quarter already affects our deviation measure $\hat{c}_{\Delta y_i}$. For the estimates in table 5 we only used data until the fourth quarter of 2019 in the construction of $\hat{c}_{\Delta y_i}$. Another hint at why it makes sense to deal with the Covid-19 impacted quarters separately.

There is a lot that has to be accounted for when trying to address this decay properly. We will start by looking at the corresponding t-statistics as well as the adjusted $R^2$ of the quarterly updated forecast regressions in figure 4. Both curves show a very similar tendency, namely, after taking some time to adjust and generate proper estimates, they perform well from the late 80s to around 2010. Then, the same decline sets in until the Covid-19 quarters erase any relevance. The goal of this section is to investigate the decay after the financial crisis and prior to the Covid-19 pandemic. The impact of the latter will then be discussed in section 5.4.
Figure 4: Newey and West (1987) corrected t-statistics and adjusted $R^2$ of quarterly-ahead forecast regressions of excess returns on $\hat{cay}_t$. The light-blue dashed lines represent significance levels of 10%, 5%, 1%, and 0.1%, respectively.

So far, we have seen that the coefficient estimate for $\hat{cay}_t$ declined over the last decade in absolute terms. However, we are more interested in changes relative to a one standard deviation increase in $\hat{cay}_t$. Of course, if the standard deviation were to stay remotely the same over the course of the period in question, there is no difference between looking at absolute and relative terms. What we do have though, is that the standard deviation of $\hat{cay}_t$ slowly decreased over decades from around 0.0187 in the late 80s to 0.0168 in 2013, just to jump back up to 0.0193 only within 2019. Thereafter, the Covid-19 observations totally shot it through the roof with up to 0.0322 lately. Thus, it makes sense to look at the predicted increase in quarterly excess returns given an increase in $\hat{cay}_t$ by one standard deviation. This is precisely what figure 5 shows us.
What we see is truly interesting. Even though the standard deviation of $\hat{c}_t\hat{y}_t$ rose in recent years, this effect does not seem to cancel out the decline in the coefficient estimate. In consequence, this means the power of $\hat{c}_t\hat{y}_t$ to predict future excess returns indeed weakened - from its peak of almost 2 percentage points per quarter in the third quarter of 2002 to the roughly 1.16 percentage points right prior to the Covid-19 pandemic we discussed before. Interestingly, the decay did not only begin from around 2010 onwards as figure 3 suggests, but in relative terms it actually already started shortly after the turn of the millennium, leading us to believe that the impact of the financial crisis on this might not be too big after all. In fact, the financial crisis seems to have shortly pushed up the relevance of $\hat{c}_t\hat{y}_t$ due to a sudden upward jump in the standard deviation of our trend deviation measure as well as a slightly higher coefficient estimate. A similar thing happened in the early 2000s, leading to those two short periods being the only time the significance level of the coefficient estimate reached the 0.1% threshold, cf. figure 4.

The according graphs for longer horizons up to one year behave very similar, albeit on different levels. For example, the adjusted $R^2$ for $H = 4$ reached its high around 15% in the third quarter of 2003, before moving downward towards the 4.3% we already saw in table 5. The plots for a one year horizon can be found in figure A.1.
5.3. The decay of $c_{day}$

So far we have only talked about $\hat{c}_{ay}_t$ in the previous section. In this section, we will now take a closer look at the (better) deviation measure $\hat{c}_{day}_t$ and its behavior over time. The regressions we ran are exactly the same but with $\hat{c}_{day}_t$ as the trend deviation measure instead of $\hat{c}_{ay}_t$. The analogue to figure 3 is given by figure 6.

Figure 6.: In green, parameter estimates for $\hat{c}_{day}_t$ in the quarterly-ahead predictions of excess returns, re-estimated each quarter from 1970:1-2022:1. The sample starts in 1952:1. The dashed black line resembles the graph for $\hat{c}_{ay}_t$ from figure 3.

Again, we see that in the early periods the estimates are varying very much due to the lack of sufficiently many observations. After that, we see an increase of the estimate from the mid-80s until the early 2000s, followed by a decline until shortly after the financial crisis. Then, after a stable period of a couple years, we see a similar but more steep linear decline as in the graph for $\hat{c}_{ay}_t$. All in all, for most periods the estimates were higher than for $\hat{c}_{ay}_t$, reaching a maximum of 1.649 in the third quarter of 2002, while the coefficient estimate for $\hat{c}_{ay}_t$ moved around one for most of the time between the late 80s and the year 2010.

Looking at the relative impact of $\hat{c}_{day}_t$ on excess returns measured for a one standard deviation increase, figure 7 shows that again the decay is indeed real. It also started shortly after the turn of the millennium and continued, albeit with a bit more variation, until the beginning of
the Covid-19 pandemic. Since then, the impact has vanished completely. What strikes the eye is that seemingly $\hat{cay}_t$ had a stronger impact than

![Figure 7](image)

**Figure 7.** In green, the predicted change in one-quarter ahead excess returns given a one standard deviation increase in $\hat{cday}_t$. The sample starts in 1952:1. The dashed black line resembles the graph for $\hat{cay}_t$ from figure 5.

$\hat{cday}_t$ for the time spanning most of the decades of the 1980s and 90s. That is in line with our findings regarding the significance of the estimates for $\hat{cday}_t$, which took until the mid-90s to reach the 1% level. In comparison to that, figure 4 shows that $\hat{cay}_t$ reached this level already in the late 80s.

In addition to that, we also want to compare the share of variation that can be explained by our forecast regressions. This is done in figure 8. Here, we also clearly see the better performance of $\hat{cay}_t$ in the past, namely the 80s and much of the 90s, before $\hat{cday}_t$ became the stronger deviation measure in the late 90s. What is interesting is that contrary to what we found for $\hat{cay}_t$ above, the financial crisis led to a huge drop in the goodness of fit for $\hat{cday}_t$. After a swift recovery, the model continued its overall decay.

All in all, it can be said that once the coefficients for $\hat{cday}_t$ reached a certain level of significance, they perform better than the ones for $\hat{cay}_t$ in terms of t-statistics as well as the adjusted $R^2$.

Another interesting feature of $\hat{cday}_t$ as our trend deviation measure is that its standard deviation steadily increased over time, from around
Figure 8.: In green, adjusted $R^2$ for the regressions using $\widehat{cday}_t$ as the only explanatory variable. The dashed black line resembles the graph for $\widehat{ca}_t$ from figure 4.

0.011 around 1980 to about 0.0186 before the Covid-19 pandemic. Lastly, there is a noticeable difference when looking at longer horizon predictions. For $H = 4$, we get a steep increase up to t-statistics of almost 5.5 from the late 70s to the second quarter of 2016, followed by a rapid drop-off into irrelevance culminating during the Covid-19 pandemic, cf. figure 9.
5.4. The impact of the Covid-19 pandemic

We have already mentioned and seen the effects of the Covid-19 pandemic on our trend deviation measures and, therefore, the forecast regressions of excess returns in the previous sections. The goal of this section is to dig a little deeper into what exactly causes these disturbances. For that, we start with having a look at the first differences in our log consumption and log income time series in figure 10. Both exhibit huge spikes never seen before, with amplitudes more than three times as large as the previous highs, in the early quarters of the pandemic. Consumption drops massively due to the lockdown measures in the U.S., especially in the second quarter of 2020. This disruption is accompanied by a similarly tremendous upward jump in income due to the transfer payments coming along with the lockdown. Both series bounce back equally strong the quarters thereafter with income exhibiting further large spikes.

When looking at changes in the asset wealth time series, the impact of Covid-19 is visible in both the aggregate asset wealth and the financial wealth time series, but its magnitude is not considerably larger than that of other stock market crashes nor rallies thereafter. The housing wealth
Figure 10.: Consumption growth in green and labor income growth in blue. Sample period is 1952:2-2022:1.

Time series does not seem to be affected at all. Appendix A.6 also provides these graphs in figure A.2. All this leads us to believe that it is the massive deviation in consumption and income from the long-run equilibrium that causes the disturbances seen and mentioned before. A drop this large in consumption alone would be enough to reduce the consumption-wealth ratio by a fairly high margin. Together with the opposite (upward) deviation in income, and, therefore, wealth, the whole ratio plummets to new lows not seen before. Figure 11 shows that formidably.
The estimated trend deviation measure $\hat{cay_t}$ for the sample period 1952:1-2022:1. The green part of the curve represents pre-pandemic observations until 2019:4. The red part shows the development during the Covid-19 pandemic. The solid blue line marks the mean of the pre-pandemic observations, accompanied by its 95% confidence interval as the dashed lines. Here, leads/lags up to $k = 8$ were used.

According to our theory laid out in chapter 2, cf. equation 2.6 again, this would lead us to expect extremely low returns in the future. For example, let us take the coefficient of 0.579 from table 5 as our estimate for the impact of $\hat{cay}_t$ on one-quarter ahead excess returns. Then, together with the intercept of 0.266 and an estimated $\hat{cay}_t$ of -0.533 in the second quarter of 2020, when the impact hit the most, we would predict an excess return in the subsequent quarter of -0.042 or roughly minus 4.12%. Now, this may not seem too bad, given that the worst quarter in the whole sample, Q3 of 1974, had an excess return of minus 26.73%. However, the next closest prediction prior is at minus 1.4% and the median is at 1.53%. Note that to be consistent with the previous results from section 5.1 we only used leads/lags up to $k = 1$ for computing the value of -0.533 for $\hat{cay}_t$. In figure 11 however, we do use $k = 8$ based on a model selection process using information criteria. Figure 11 also shows that the only other time $\hat{cay}_t$ left its 95% confidence bandwidth before was during the early 70s, and, therefore, around the time of the oil crisis and the aforementioned worst quarter in the sample in terms of excess returns. A behavior we already saw in figure 2. Still,
this was nothing compared to the impact the Covid-19 pandemic had on our trend deviation measure. In this particular case, we are only showing the graph for $\hat{cay}_t$ but the results do not substantially differ for $cday_t$.

To further illustrate how much of an outlier the observations from quarters during the pandemic are, we plotted excess returns against the one-quarter lagged values of $\hat{cay}_t$ that the excess returns are regressed on in figure [12].

![Figure 12](image)

**Figure 12.** Excess returns plotted against the one-quarter lagged values of $\hat{cay}_t$. Green circles represent observations from 1952:1-2019:3. The yellow triangle resembles the fourth quarter of 2019 together with the excess return of 2020:1. The red crosses represent the eight observations during which the pandemic was already affecting $\hat{cay}_t$ from 2020:1-2021:4.

It shows perfectly why the linear regressions using the Covid-19 impacted observations break down, leading to negligible coefficient estimates as well as a practically nonexistent $R^2$ statistics.

A natural question to ask is of course whether consumption and labor income will converge back towards their long-run equilibrium with asset wealth. The most recent observation from the first quarter of 2022 suggests so but is of course only one step into that direction. For us to say that the heavily impacted Covid-19 quarters were indeed sheer outliers, and to then probably just exclude them from the estimation of the long-run trend relationship, we need a consistent reversion to the long-run trend we have observed over decades before. Assuming this happens, de-
spite ongoing uncertainties surrounding the further development of the pandemic as well as other macroeconomic risk factors such as the high inflation rates and the trend towards deglobalization as a result of intensifying political tensions between different parts of the world, we can then analyze whether $\hat{cay}_t$ and $\hat{cday}_t$ bounce back to their pre-pandemic levels. In a next step we can ask if they continue their decay from before, that we have exhibited in the previous two sections. To get to the bottom of that decay and find out where it came from in the first place, remains an open task for academic research. We will provide one possible answer to that in the following chapter, building upon the work of Hahn and Lee (2006).
As usual when dealing with prediction models, we want to undertake some robustness checks when it comes to out-of-sample forecasting rather than in-sample as we have seen so far. For simplicity, we only focus on $cay$ in this chapter but the theory as well as the results also apply for anything that is related to $cday$.

In their original paper, Lettau and Ludvigson (2001) mention that $\hat{cay}_t$ as a predictor for future stock market returns is subject to a certain “look-ahead bias” due to the fact that the cointegration parameters used to construct $\hat{cay}_t$ are estimated using the full sample. By that, information that is not yet available at a certain point in time does factor into the forecast regressions for that period nevertheless, leading to a “bias” in said forecast regressions. To show how this implicit involvement of a time trend distorts the results produced by $\hat{cay}_t$, Brennan and Xia (2005) introduce a variable named $\hat{tay}_t$, that is nothing but the residual of the same cointegration regression just with consumption being replaced by calendar time $t$. They then show that the predictive abilities of $\hat{tay}_t$ are at least as good and sometimes even better than that of $\hat{cay}_t$, arguing that using the simple time trend $t$ instead of consumption $c$ just represents an ex-post trend fitting and cannot have any forecasting power.

This principle idea is picked up by Hahn and Lee (2006). They, however, argue that ignoring a deterministic time trend in the estimation of the cointegrating relationship between consumption, asset wealth, and labor income leads to a biased estimate of $\hat{cay}_t$. In the presence of such a time trend, $\hat{cay}_t$ is actually a combination of the unbiased cointegration residual and a highly persistent bias component. According to their findings, any forecasting power of $\hat{cay}_t$ is mostly attributable to the latter and, therefore, shines a weak light on the robustness of the forecasting abilities of $\hat{cay}_t$. In short, they conclude that the true $\hat{cay}_t$ accounting for the deterministic time trend is the sum of the $\hat{cay}_t$ from Lettau and Ludvigson (2001) and $\hat{tay}_t$ from Brennan and Xia (2005) where the latter has to be multiplied with the coefficient estimate of the time trend from the cointegration relationship.

The effects of that on the forecasting regressions are huge. Using the method by Lettau and Ludvigson (2001), i.e., omitting a possible time trend, Hahn and Lee (2006) get an adjusted $R^2$ of over 9%. When ac-
counting for a time trend, the bias component can explain over 8% of the variation, whereas the cointegrating residual can explain only about 3.5%. This strongly suggests a much weaker forecasting ability of $\hat{cay}_t$ than what [Lettau and Ludvigson (2001)] originally presented. But how does this fit into our topic of the “decay of $cay$”? Could it be that the relevance of the time trend [Hahn and Lee (2006)] found more than 15 years ago has vanished by now? Then the decay in the coefficient estimates and adjusted $R^2$ we found in section 5.2 could at least partially be attributed to the fact that $\hat{cay}_t$ was never a strong predictor for excess stock returns in the first place, but rather that an omitted variable bias led to spurious results in the past. If the time trend does not play as much of a role anymore, it could just be that the biased results from earlier periods have simply converged to their true unbiased counterparts, seemingly showing a decay that never actually happened. It was rather the huge impact of the bias component in the past that made results appear stronger then they actually were. Following this line of reasoning, it is of the utmost importance to test for a deterministic time trend in our cointegration relationship just as [Hahn and Lee (2006)] do. The results on that are quite clear. When employing a DLS specification using leads/lags of up to $k = 8$ to estimate the cointegration relationship of consumption, asset wealth, and labor income, the estimate for the time trend is nonsignificant with a p-value of almost 0.7, lending great power to the above reasoning into what could have caused or at least intensified the “decay of $cay$”. With these findings, the quotation marks around that term seem to be somewhat appropriate again, after we seemed to have found evidence to drop them, cf. figures 3 and 4. To investigate this further remains a task for future research and goes beyond the scope of this thesis. Just note that even if this truly is part of the explanation of the decay, it does not exclude other explanations entirely. Especially changes in monetary policy substantially influencing the inner workings of financial markets may play a substantial role in the “decay of $cay$”.

Lastly, we want to present out-of-sample performances of $\hat{cay}_t$. For that, we recursively added one quarter of observations starting again in the first quarter of 1952. When running our forecast regressions, we only used information available at that certain point in time for both the cointegration estimation as well as the actual stock return forecast. This procedure guarantees a true ex-ante estimation of $cay$ compared to the ex-post version including the “look-ahead bias” from [Lettau and Ludvigson (2001)]. Afterwards, we apply the typical loss function of squared residual errors to compare the performance to a prevailing mean model that uses nothing but the current mean as a predictor for next quar-
ters stock market returns. For the in-sample forecasts the benchmark model we are evaluating against is the full sample mean. The results are shown in figure 13 below. We closely followed Welch and Goyal (2008) in plotting the cumulative sum of squared errors (SSE) difference between our models and their respective benchmark. We also started the out-of-sample forecasting period in 1965, and thereby, 13 years after the in-sample forecasts starting in 1952.

Figure 13.: The graph shows the in-sample performance of $\hat{cay}$, as the dashed line starting in 1952 as well as the out-of-sample performance as the solid black line from 1965 onwards. The confidence band of the out-of-sample forecast in blue is based on 95\% two-sided OOS-t critical values from McCracken (2007). The sample period is 1952:1-2019:4.

Similar to what Welch and Goyal (2008) found back in 2008, we see that the positive in-sample performance mainly comes from around the time of the oil crisis in the 1970s. Thereafter, the in-sample performance has not changed substantially and even decreased a little most recently. On the other hand, the out-of-sample forecasts never performed particularly well and saw a sharp drop by the time of the 1987 stock market crash. The performance thereafter was a constant up and down until a strong decline set in after the financial crisis of 2007/08. We can conclude that, all in all, the out-of-sample performance of $cay$ is pretty weak and does not show a great amount of robustness.
7. Conclusion

We have established a theoretical framework in which the consumption-wealth ratio should be able to forecast stock market returns. Building on that, we used proxies for consumption and the unobservable component of wealth, human capital, to estimate a long-run trend relationship between consumption, asset wealth, and labor income. Deviations from this shared trend are captured in our variables \( cay \) and \( cday \), where the latter splits up asset wealth into financial and housing wealth, trying to capture transitory movements to adjust back to the long-term equilibrium more accurately.

We showed that both variables exhibit forecasting power for the sample period of 1952 until right before the beginning of the Covid-19 pandemic. This forecasting power, however, seems to have weakened over the last two decades in relative terms, raising the question of what caused this “decay of \( cay \)”. A possible explanation of that is given by the negligence of a deterministic time trend inherent in the cointegrating relationship in the past, causing biased results. The absence of such a time trend in our recent sample suggests that the “decay of \( cay \)” could just be a consequence of the vanishing of said time trend. Further research will have to gain more clarity on that matter.

We also looked at the impact of the Covid-19 pandemic on our trend deviation measures, showing a massive drop due to consumption breaking away whilst wealth was cushioned by transfer payments. Here, it remains to be seen in future observations whether the long-run relationship can be restored in order to gain back the forecasting power we observed until the beginning of the pandemic.

The fact that in true out-of-sample forecasts, where \( cay \) does not contain any “look-ahead bias”, \( cay \) was not able to beat a prevailing mean model in terms of squared error deviance generally casts doubt on the ability of the consumption-wealth ratio to forecast stock market returns - a feature \( cay \) has in common with other famous forecasting variables such as the earnings-price ratio, the dividend yield, or the dividend-payout ratio.
A. Appendix

A.1. Data description

Consumption. Consumption is defined as total personal consumption expenditures (PCE) as already discussed in section 3.2. Data are quarterly, seasonally adjusted at an annual rate, measured in billions of dollars (2012 prices), in per capita terms and expressed in the logarithmic form. Series comprises the period 1947:Q1–2022:Q1. The source is the U.S. Bureau of Economic Analysis, NIPA Table 2.3.5 line 1, retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/PCEC, July 15, 2022.


**Housing wealth.** Housing wealth is defined as the value of real estate held by households (line 4 of Table B.101 — series LM155035015.Q) minus home mortgages (line 33 of Table B.101 — series FL153165105.Q). Data are quarterly, end of period, measured in billions of dollars (2012 prices), in per capita terms and expressed in the logarithmic form. Series comprise the period 1951:4–2022:1. The source is the Board of Governors of the Federal Reserve System, Financial Accounts, Table B.101, retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/release/tables?rid=52&eid=810090, July 15, 2022.

**Labor income.** Labor income is defined as the sum of wages and salaries (line 3), personal current transfer receipts (line 16), and employer contributions for employee pension and insurance funds (line 7) minus employee contributions for government social insurance, and taxes. Employee contributions for government social insurance are defined as personal contributions for government social insurance (line 25) minus employer contributions for government social insurance (line 8). Taxes are defined as \[\frac{\text{wages and salaries (line 3)}}{\text{wages and salaries (line 3)} + \text{proprietor’s income with inventory valuation and capital consumption adjustments (line 9)} + \text{rental income of persons with capital consumption adjustment (line 12)} + \text{personal dividend income (line 15)} + \text{personal interest income (line 14)}}\] * (personal current taxes (line 26)). Data are quarterly, seasonally adjusted at annual rates, measured in billions of dollars (2012 prices), in per capita terms and expressed in the logarithmic form. Series comprise the period 1947:1–2022:1. The source is the U.S. Bureau of Economic Analysis, NIPA Table 2.1, retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/release/tables?rid=53&eid=4083, July 15, 2022.

**Population.** Population is defined by dividing aggregate real disposable income (line 37) by per capita real disposable income (line 39). Data are quarterly. Series comprises the period 1947:1–2022:1. The source is the U.S. Bureau of Economic Analysis, NIPA Table 2.1, retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/release/tables?rid=53&eid=4083, July 15, 2022.

**Price deflator.** Consumption, nominal wealth, and labor income were deflated by the personal consumption expenditure chain-type price deflator (2012=100), seasonally adjusted. Data are quarterly. Series comprises the period 1947:1–2022:1. The source is the U.S. Bureau of Economic Analysis, NIPA Table 2.3.4 line 1, retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/release/tables?rid=53&eid=4083, July 15, 2022.

**Inflation rate.** The inflation rate is computed from the price deflator. Data are quarterly. Series comprises the period 1947:2–2022:1.

**Interest rate (“Risk-free” rate).** The “risk-free” rate is defined as the 3-month U.S. Treasury bills real interest rate per quarter. Original data are monthly nominal rates in percent per annum and are converted to a quarterly frequency by computing the simple arithmetic average of three consecutive months and applying the discount method. The real interest rates are computed as the difference between nominal interest rates and the inflation rate. The 3-month U.S. Treasury bills real interest rate’s series comprises the period 1947:2–2022:1. The source of the nominal interest rate’s series is the H.15 publication of the Board of Governors of the Federal Reserve System, retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/TB3MS, July 15, 2022.

**Asset returns.** Asset returns are computed using the CRSP Value-Weighted Total Return Index for the S&P 500 Universe. It measures market performance assuming a full reinvestment of all distributions. The original series is daily. Monthly returns are computed using the data from the last business day of each quarter. The series comprises the period 1926:1-2022:1.
A.2. Cointegration tests

Recall from the theory laid out in chapter 2 that the whole estimation of $\hat{cay}_t$ and $cday_t$ in chapter 3 is based on the assumption that there is a cointegrating relationship between the respective variables used to construct these measures. We again follow Lettau and Ludvigson (2001) in applying the procedure suggested by Johansen (1991) in order to test not only for a cointegrating relationship in general but also the number of such if any are given. We also do allow for linear trends in the data, but assume that the cointegrating relation only has a constant and no deterministic trend. Especially the last assumption has to be seen from a critical point of view given the findings by Hahn and Lee (2006). Note that the Johansen (1991) procedure requires $I(1)$-time series, something we have tested for already in section 3.3. Table A.1 presents the results for consumption, asset holdings, and labor income from several such Johansen (1991) tests using both the eigenvalue and trace type, as well as different lags.

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<td>10.198</td>
<td>13.75</td>
<td>15.22</td>
</tr>
<tr>
<td>5.05</td>
<td>7.52</td>
<td>5.05</td>
</tr>
</tbody>
</table>

Table A.1: Results from various Johansen (1991) test procedures. The columns labeled “Test Statistic” give the value for the test named in the row above while the columns labeled “90% CV” give the 90 percent confidence level of that statistic.

Among both types of tests and all different lags, we can always reject the null hypothesis of no cointegration. At the same time, we can never reject the null of only one cointegrating relationship versus the alternative of two according to the test statistics from the L-Max procedure, nor
the null of one cointegrating relationship against the alternative of three according to the test statistics from the trace procedure.

The equivalent results for the cointegrating relationship between consumption, financial asset holdings, housing asset holdings, and labor income, can be found in table A.2.

<table>
<thead>
<tr>
<th></th>
<th>L-Max</th>
<th>Trace</th>
<th>$H_0 = r$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Test Statistic 90% CV</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel A: Lag in VAR Model equal to 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>52.10</td>
<td>25.56</td>
<td>74.22</td>
<td>49.65</td>
</tr>
<tr>
<td>11.79</td>
<td>19.77</td>
<td>22.12</td>
<td>32.00</td>
</tr>
<tr>
<td>6.32</td>
<td>13.75</td>
<td>10.34</td>
<td>17.85</td>
</tr>
<tr>
<td>4.02</td>
<td>7.52</td>
<td>4.02</td>
<td>7.52</td>
</tr>
<tr>
<td>Panel B: Lag in VAR Model equal to 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>38.38</td>
<td>25.56</td>
<td>63.26</td>
<td>49.65</td>
</tr>
<tr>
<td>13.45</td>
<td>19.77</td>
<td>24.89</td>
<td>32.00</td>
</tr>
<tr>
<td>6.81</td>
<td>13.75</td>
<td>11.43</td>
<td>17.85</td>
</tr>
<tr>
<td>4.62</td>
<td>7.52</td>
<td>4.62</td>
<td>7.52</td>
</tr>
<tr>
<td>Panel C: Lag in VAR Model equal to 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33.14</td>
<td>25.56</td>
<td>61.84</td>
<td>49.65</td>
</tr>
<tr>
<td>12.52</td>
<td>19.77</td>
<td>28.71</td>
<td>32.00</td>
</tr>
<tr>
<td>11.19</td>
<td>13.75</td>
<td>16.19</td>
<td>17.85</td>
</tr>
<tr>
<td>5.00</td>
<td>7.52</td>
<td>5.00</td>
<td>7.52</td>
</tr>
</tbody>
</table>

Table A.2: Results from various [Johansen (1991)] test procedures. The columns labeled “Test Statistic” give the value for the test named in the row above while the columns labeled “90% CV” give the 90 percent confidence level of that statistic.

Again, the null of no cointegration can always be rejected for both types and all lags. At the same time, we never reject the null of one cointegrating relationship against the respective alternative for both types of tests.

Hence, we may conclude that there seems to be a single cointegrating relationship between consumption, asset holdings, and labor income as well as consumption, financial asset holdings, housing asset holdings, and labor income. This strongly justifies the estimation procedure we conducted in chapter 3.
A.3. Long horizon forecasts

In section 5.1 we showed tables for $H$-period forecast regressions of excess returns on the one-period lagged value of our deviation measures $\hat{cay}_t$ and $\hat{cday}_t$ for horizons up to only $H = 4$, i.e., one year. The following tables show the results for one quarter ahead predictions together with horizons from one year up to five years in yearly steps.

Table A.3.: $H$-period excess return forecast regression estimates for $\hat{cay}_t$ and $\hat{cday}_t$. Newey and West (1987) corrected t-statistics appear in parenthesis. The sample period is 1952:1-2019:4 for $\hat{cay}_t$ and $\hat{cday}_t$, 1952:1-2020:1 for excess returns, respectively.
### A.4. The consumption growth forecast regressions

<table>
<thead>
<tr>
<th>$H$</th>
<th>Dependent variable: $\Delta c_{t+1} + \ldots + \Delta c_{t+H}$</th>
<th>Adjusted $R^2$</th>
<th>Adjusted $R^2$</th>
<th>Note:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\hat{c}_{\text{ay}}_t$</td>
<td>-0.001</td>
<td>-0.001</td>
<td><em>p &lt; 0.1</em></td>
</tr>
<tr>
<td></td>
<td>(0.088)</td>
<td>(0.076)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(Intercept)</td>
<td>-0.002</td>
<td>-0.002</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$\hat{c}_{\text{ay}}_t$</td>
<td>-0.024</td>
<td>-0.024</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.686)</td>
<td>(0.374)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(Intercept)</td>
<td>-0.001</td>
<td>-0.001</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$\hat{c}_{\text{ay}}_t$</td>
<td>-0.015</td>
<td>-0.015</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.169)</td>
<td>(0.239)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>(Intercept)</td>
<td>-0.001</td>
<td>-0.001</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>$\hat{c}_{\text{ay}}_t$</td>
<td>-0.020</td>
<td>-0.020</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.237)</td>
<td>(0.402)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>(Intercept)</td>
<td>-0.006</td>
<td>-0.006</td>
<td></td>
</tr>
</tbody>
</table>

Note: $^*$ $p < 0.1$; $^*^*$ $p < 0.05$; $^*^*^*$ $p < 0.01$

Table A.4.: $H$-period consumption growth forecast regression estimates for $\hat{c}_{\text{ay}}_t$ and $\hat{c}_{\text{day}}_t$. Newey and West (1987) corrected t-statistics appear in parenthesis. The sample period is 1952:1-2019:4.
A.5. The decay of $cay$ and $cday$ for longer horizon forecasts

At the end of section 5.2, we mentioned that the graphs of the longer horizon excess forecasts using $\hat{cay}_t$ behave similar to the graphs we showed for one-quarter ahead forecasts. As an example, we want to give the respective graphs for the 4-period excess return forecasts, i.e., one-year ahead forecasts. Figure A.1 shows the graph for the estimates, their corresponding $t$-statistics and the adjusted $R^2$.

Figure A.1.: The three graphs show from top to bottom:
1) The estimates for $\hat{cay}_t$ in 4-period excess return forecasts.
2) The corresponding Newey and West (1987) corrected $t$-statistics including confidence levels of 10%, 5%, 1%, and 0.1%, respectively.
3) The corresponding adjusted $R^2$.
A.6. The asset wealth growth series

In section 5.4 we pointed out that the Covid-19 pandemic massively affected our consumption growth and labor income growth time series, see figure 10. At the same time, aggregate asset wealth growth, as well as its components financial wealth growth and housing wealth growth, do not seem to be affected in an unusually high manner.

Figure A.2.: Aggregate asset growth in red, financial asset growth in dark-red, and housing asset growth in orange. The sample period is 1952:2-2022:1.

We do see the stock market crash of March 2020 being a noticeable event in the first two series, albeit not being significantly larger than previous drops in asset wealth. The fast rebound thereafter exhibits the largest upward jump in both series, but again, not in a way that we would label it as extraordinary compared to what we observed in figure 10. The housing wealth growth series does not seem to be affected at all by the pandemic, it was, however, largely so by the financial crisis and the burst of the housing bubble. A longer period of growth before the crisis was followed by a similarly long period of shrinking housing wealth after.
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Innsbruck, am .............. ...................................

(Unterschrift)